THE DESIGN OF MINIMUM WEIGHT GRILLAGES BY INVERSE METHODS

by

HAROLD O. BULLOCK, JR., LIEUTENANT, UNITED STATES NAVY

B.S., U.S. Naval Academy (1957)

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF NAVAL ENGINEER

AND THE DEGREE OF

MASTER OF SCIENCE IN NAVAL ARCHITECTURE

AND MARINE ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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ABSTRACT

This thesis proposes a direct, one step procedure for designing a grillage which will have minimum stiffener weight. The premise upon which this procedure is based is that the bending stress in the flange of every stiffener is constant everywhere in the grillage. This basic premise was first examined for its validity. The basic beam equations show that constant stress in the beam flanges will be achieved when the product of beam curvature and the distance from the neutral axis to the extreme fiber is constant. Two possible approaches were investigated; one where both curvatures and distance to neutral axis were constant, and a second where one was inversely proportional to the other. Design equations using the constant curvature approach were developed for a p x q grillage of rectangular plan form under uniform pressure. Both simply supported and fully clamped ends were considered. The only design decisions necessary with the proposed procedure are the choice of the design stress and the maximum allowable deflection.

Application of the proposed technique to grillages made up of plated tee beams is also developed. Design charts are included which permit the determination of stiffener sizes necessary to give the required inertia variations with constant depth to the neutral axis in accordance with the design equations. A 3 x 3 plated tee grillage is completely designed by the proposed method and compared with a similar

grillage designed by a conventional method. The stiffener weight in the proposed design is only about 50% of the stiffener weight as obtained by conventional methods. If the plating weight is also considered, the proposed grillage is about 20% lighter than the conventional design.

The practical aspects of building such a minimum weight grillage are discussed, and some general guidelines are proposed. The general conclusion is that the proposed design technique can be both practical and highly advantageous in situations where structural weight is a critical factor.

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Title: Assistant Professor of Naval Architecture (Visiting)

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Many of the theoretical calculations in this work were done at the Computation Center at M.I.T., Cambridge, Massachusetts.

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NOTATION

Symbol	Meaning
a	Spacing between longitudinal (y) beams.
A	An area - specific areas are defined by appropriate subscripts where used.
b	Spacing between transverse (x) beams.
c	Distance from neutral axis to outermost fiber of cross-section.
c_x^j	Value of c in the x direction on the j^{th} transverse beam.
c_y^i	Value of c in the y direction on the i^{th} longitudinal beam.
d	Nominal depth of plated tee section - from mid-thickness of plate to mid-thickness of flange.
d_s	Depth of tee stiffener required to satisfy shear requirements.
E	Modulus of Elasticity of material.
g	Breadth of rectangular cross section.
h	Depth of rectangular cross section.
H	Static head of salt water in feet.
i	Index in the transverse (x) direction; $-\left[\frac{p-1}{2}\right] \leqq i \leqq \left[\frac{p-1}{2}\right]$
I	Moment of inertia of beam cross-section about its neutral axis.
I'	Second moment of an area about an assumed neutral axis (XX).
I_{cg}	Moment of inertia of an area about its own neutral axis.

I_x^j Variable moment of inertia of the j^{th} transverse in the x direction.

I_y^i Variable moment of inertia of the i^{th} longitudinal in the y direction.

j Index in the longitudinal direction;

$$-\left[\frac{q-1}{2}\right] \leqq j \leqq \left[\frac{q-1}{2}\right]$$

M Bending moment along a beam.

M_x^j Bending moment in the x direction on the j^{th} transverse.

M_y^i Bending moment in the y direction on the i^{th} longitudinal.

p Number of longitudinal (y) beams.

P Uniform hydrostatic pressure.

q Number of transverse (x) beams.

Q_x Distributed line load on the transverse (x) beams.

Q_y Distributed line load on the longitudinal (y) beams.

r Ratio of nominal depth of plated tee to the flange width, i.e. $r = \frac{d}{w_f}$

R An internal reaction between intersecting beams of a grillage, positive downward.

R_j^i The internal reaction <u>on</u> the j^{th} transverse due to the intersection with the i^{th} longitudinal.

R_i^j The internal reaction <u>on</u> the i^{th} longitudinal due to the intersection with the j^{th} transverse.

t Plating thickness.

t_f Flange thickness of tee stiffener.

t_w Web thickness of tee stiffener.

U	Strain energy function.
V	End support reactions, positive upward.
w	Deflection in the Z direction; positive upward.
w_o	Deflection of the origin; maximum deflection.
w_c	Deflection in the central portion of a fully clamped beam.
w_e	Deflection in the end portion of a fully clamped beam.
w_x^j	Deflection of the j^{th} transverse in the x direction.
w_y^i	Deflection of the i^{th} longitudinal in the y direction.
$w_x^{j''}$	Second derivative (curvature) of the deflection of the j^{th} transverse with respect to distance (x).
$w_y^{i''}$	Second derivative (curvature) of the deflection of the i^{th} longitudinal with respect to distance (y).
x	Distance from origin (center of grillage) in transverse direction.
y	Distance from origin (center of grillage) in longitudinal direction.
Z	Section modulus of cross-section; i.e. $Z = \frac{I}{c}$
σ	Design bending stress.
τ	Design shear stress.
γ	Specific weight of material.
$'$	First derivative with respect to length.
$''$	Second derivative with respect to length.

I. INTRODUCTION

In a given structure under load much of the material which is stressed to a level less than the design limit is redundant. The redundancy arises since the strength contribution of this understressed material is small compared to the weight which it adds to the structure. This fact has long been recognized and many methods have been used to eliminate redundant material. Some rather simple structures such as beams have been designed to give a lower weight of structure than conventional designs by the use of cover plates in locations where greater inertia is required to resist bending. Another technique is the judicious use of lightening holes. Ideally, in a simple beam the section modulus should be a constant fraction of the bending moment. However, for a highly redundant structure, such as a grillage, the mechanics of continuously relating the required moment of inertia of the section, or its section modulus, to the applied bending moment is a rather formidable obstacle to the development of minimum weight designs.

Until the present time the general design procedure for many redundant structures has been to select the sizes of the structural elements rather arbitrarily. The resulting maximum stresses and deflections due to the applied loading are then calculated. These are either accepted or the structure modified and new cal-

culations made. Such techniques merely ensure that the prescribed stress is nowhere exceeded. Most of the structure is stressed below its design limit. It is apparent that conventional methods of grillage design must lead to considerable unnecessary weight. Conventional techniques merely produce an adequate structure for the prescribed conditions with no indication of how good the structure really is from the viewpoint of structural weight. In shipbuilding structural weight is an extremely important factor since it has great influence on how much payload or armament can be carried. Thus, minimizing structural weight if of great interest to the shipbuilder.

The object of this thesis is to unite two separate ideas put forth by Heyman [1], [2].* The desired result is one design procedure which will produce a structure of absolute minimum weight. One of Heyman's two proposals was to assume a deflected form of a beam or grillage. By double differentiation the curvatures could then be determined. Normal beam theory then gave the required inertia as

$$I = \frac{M}{E\,w''} \qquad (1)$$

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* Numbers in brackets [ ] refer to references in the bibliography

a function of the curvature and the loads. This technique is termed an inverse design method in that it is the inverse of the normal design procedure. The normal procedure is to start with an inertia variation, usually constant, and by double integration determine the deflection.

Heyman's second proposal was that to achieve absolute minimum weight, the necessary condition was that the deflected shape of the structure must consist of arcs of constant curvature, with no discontinuity of slope where two arcs join. Thus, by applying the minimum weight requirement of constant curvature to the inverse design procedure as a constraint, the resulting design not only gives the scantling sizes directly, but it also gives minimum weight at the same time. Although the redundant reactions must still be considered in the analysis it will be shown that with this procedure these reactions are purely arbitrary and may take any prescribed values without affecting the weight of structure, which will be a minimum.

The criterion of constant curvature established by Heyman for absolute minimum weight holds specifically for the beams of symmetric cross section and constant depth which he considered. The beam equations show that constant curvature and constant c

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will indeed produce constant σ, because

$$\sigma = \frac{M c}{I} = E c w'' \qquad (2)$$

However, if the criterion considered is simply constant stress in the outer fiber at all points along the length, any combination of c and w" such that their product is constant, will produce the desired result. For grillages as found in a ship's bottom, where the flange consisting of the bottom plating is of a constant and rather large area, the symmetry of flange variation used by Heyman is not available. Only the stiffener depth and flange width can be varied to achieve the required inertia variation. However, since constant stress also does not necessarily imply constant c, any desired curvature can be achieved by properly varying c, the depth to the neutral axis. It will be shown that for simplicity of analysis constant curvature is still the most desirable technique for plated tee beams.

A technique for determining the required stiffener variations for some typical plating sizes will be developed. Examples of designs using both constant and varying curvature are included. In both cases weight savings of up to 100%, neglecting shear requirements, are achieved.

II. PROCEDURE

For the development of the design procedure, the following assumptions were made at the outset:

1. The material obeys Hooke's law at all times.
2. Small deflection, linearized beam theory is applicable.
3. For plated grillages with tee stiffeners, the effective breadth of plating remains constant along the length of of any stiffener.
4. Torsional effects are small enough to be neglected.
5. Shear deflections are neglected.
6. Plate thickness is determined from any standard plating design procedure, which is not considered as an integral part of the grillage design procedure developed in this report.
7. Plate and stiffeners have the same modulus of elasticity.

The effects of shear and end loads are not integrated into the design procedure. However, their possible effects are discussed in Section IV. In addition, all grillages considered consisted of an odd number of beams in each direction uniformly spaced. The procedure developed is applicable only to flat grillages having rectangular plan form

The first step in the de-

sign procedure is to assume a deflected shape for the loaded grillage. The choice of a deflected shape controls or is controlled by the type of design procedure to be followed, as discussed below. Once a deflected shape is selected, the curvatures, w", along the stiffeners can be determined. Then, from simple beam theory, we know that

$$\sigma = E\,c\,w'' = \frac{M\,c}{I} = \frac{M}{Z} \tag{3}$$

in which σ is the bending stress in the extreme fiber, a distance c from the neutral axis. Now, for absolute minimum weight, it appears that the condition that the bending stress (σ) be constant everywhere is the only criterion which need be satisfied. Heyman has shown [1] that for c = Constant, absolute minimum weight is achieved when w", the curvature, is also constant. However, the results in Section III suggest that these restrictions on c and w" are not really necessary, and that σ = constant is the only criterion required for absolute minimum weight.

Accepting this basic hypothesis that constant stress everywhere in the extreme fiber corresponds to absolute minimum weight, there are obviously two approaches one can take to the design problem. First, following Heyman's approach take w" as constant. Then, the deflected shape must necessarily be of a form which gives constant curvature along any particular stiffener. The most simple

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deflected shape, and the one utilized in this thesis, is a second order polynomial. The second alternative is not to restrict c and w'', in which case, from equation (3) we see that $c \sim \frac{1}{w''}$ and $Z = I/c \sim M$. Since the curvature cannot be constant in this case, it would appear that any deflected shape giving a non-constant curvature might be used. However, examination of the various possible deflected forms indicates that, for simply-supported grillages, a polynomial of order four or greater is necessary to prevent c from being indeterminate.* (Due to the choice of the origin at the center of the grid, symmetry requirements eliminate all odd function terms in the deflected shapes).

The second alternative mentioned above leads to very complicated mathematics, and will not be considered further here. A procedure using a 6^{th} order polynomial is worked out in Appendix A to illustrate the techniques and some of the problems involved.

The detailed procedure developed by the authors uses the first approach of keeping c and w'' constant in order to achieve constant stress in the extreme fiber everywhere in the grillage.

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* It is necessary to include the second order term since it is this term which prevents c from being indeterminate.

The detailed steps in the procedure are:

<u>Step 1</u>: Assume a deflected shape and determine the curvatures in the x and y directions. The assumed shape must result in constant curvature along each beam.

<u>Step 2</u>: From the equilibrium conditions, determine the expression for the bending moment along any beam in terms of the applied load on the beam, Q, and the internal reactions R, which exist at each intersection along the beam.

<u>Step 3</u>: Relate the moment and curvature expressions according to

$$M = E\,I\,w''$$

and then solve for I

$$I = \frac{M}{E\,w''}$$

<u>Step 4</u>: Relate stress to curvature through the equation

$$\sigma = E\,c\,w''$$

and thereby determine the required value of $c = \dfrac{\sigma}{E\,w''}$.

The resulting equation for I will give the variation in inertia required to keep the maximum allowable bending stress constant throughout the length of the beam, with the restriction that c remain constant along the length.

<u>Step5</u>: Knowing c, and the required variation of I along the beam, the actual scantlings can then be determined. The method used to determine the actual scantlings will de-

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pend on the cross-sectional shape to be used. For rectangular cross sections, the proper dimensions of the section can be determined analytically. For plated tee sections, the required scantlings may be obtained from design charts, which is the method used in this thesis. General expressions for the moment of inertia and distance from the neutral axis to the extreme fiber will be developed for plated tee sections of the type used in ship construction. From these expressions, curves relating I and c will be constructed for different thickness of plating and varying stiffener proportions. In addition, cross curves will also be constructed to allow determination of the actual stiffener scantlings for any combination of I and c.

III RESULTS AND DISCUSSION

While exploring the various aspects of the minimum weight design procedure, several steps were taken to examine different aspects in detail. Specifically, the following separate investigations were made.

1. A simply supported beam, consisting of a tee bar on plating, was designed by three different methods. This was done to test the validity of the argument that constant stress in the extreme fiber is the sole criterion for absolute minimum weight.

2. Using constant curvature and constant c as the design criteria, an inverse design procedure for gillages was developed. This procedure is applicable to a p x q grillage under uniform pressure with an odd number of uniformly spaced beams in each direction. Both simply supported and fully clamped ends were considered.

3. By considering beams with rectangular cross sections, a closed form of the weight equation for a p x q grillage was obtained. Further investigation showed that for beams with symmetric cross section, the total stiffener weight is independent of the values of the internal reactions between the beams in the grillage.

4. Geometric properties of plated tee beams were computed and the results plotted in Figures VII through XXII. Using these curves, a sample 3 x 3 grillage was designed utilizing the inverse design procedure. The total stiffener weight was determined and compared with the stiffener weight of a conventional design.

1. Simply Supported Beam

Figure XXIII of Appendix C (1) shows the loading configuration for the three beam types discussed below. The results of simple beam design by the three different methods are as follows:

Design Method	Beam Weight (lbs)
a. Conventional, prismatic members	724.4
b. Constant curvature, constant σ	287.5
c. Non-constant curvature, constant σ	304.5

As previously stated, the purpose of this exercise was to see if σ = constant everywhere in the extreme fiber was a sufficient criterion for absolute minimum weight. The details of these designs are presented in Appendix C (1). For the constant stress designs (b) and (c), shear considerations determined the cross sectional size of the beam at and near the supports, since at the support the section need have no inertia according to the pure bending requirements, and near the support, the required inertia is quite small.

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While the determination of an absolute minimum is very straightforward in mathematical theory, practical considerations often prevent a rigorous mathematical approach. Intuition suggests that structural weight will be a minimum when all the available material in the extreme fiber is stressed to its design limit. For simple geometries such as rectangular sections a mathematical approach to the minimization problem is not too difficult. However, for practical structural shapes such as plated tee beams a rigorous mathematical approach is not feasible, and empirical relations and approximations must be used.

Ideally, if constant stress in the extreme fiber were the only criterion for absolute minimum weight, designs (b) and (c) should have exactly the same weight and it should be less than the weight of a beam obtained by any other design method not satisfying the constant stress criterion. The results show that while they are not quite equal, the weights of (b) and (c) are considerably less than the weight of the corresponding prismatic beam. It is the opinion of the authors that within the accuracy of the procedures used to obtain designs (b) and (c), their weights are close enough to be considered equal, and thereby tentatively substantiate the premise that absolute minimum weight will be achieved if the ex-

treme fiber stress is constant throughout the structure. This is a considerable extension of the absolute minimum weight criterion proposed by Heyman [1] . Since Heyman was only concerned with beams of constant overall depth and symmetric flanges the value of c for a given beam was constant. When the deflection was constrained to be second order, the beam equations (2) show that constant stress in the extreme fiber resulted. It is suggested here that the constant stress criterion is the true requisite, and any relation between c and curvature which produces constant extreme fiber stress automatically ensures minimum structural weight.

Throughout this design procedure, the extreme fiber stress, σ, is purely a bending stress. The introduction of shear requirements is then a practical constraint which is not considered in the minimization process as such, and, consequently, the minimum achieved can no longer be precisely the absolute minimum. However, it is felt that the deviation from the absolute minimum will be very small due to the consideration of shear.

It should be noted that in the non-constant curvature design (c), the design parameters exceeded the limits of the design curves, and, consequently, the cross section dimensions at x = 7.5 feet were estimated based on the shear requirements

at the end. It is felt that this rather gross approximation is basically responsible for the difference in weight of the two constant stress designs. The other source of error occurs when interpolating on the log scale in the design curves. Therefore, the authors feel that the results are sufficiently accurate to substantiate the argument that σ = constant everywhere in the extreme fiber is sufficient for absolute minimum weight.

The other obvious result of these three designs is that the non-constant curvature technique is not very practical. As seen from design (c), the variation in depth is quite large, and well beyond the range of practical structures. It is possible that a more reasonable structural shape might be obtained with a higher order deflected shape, but this appears to be of questionable value since the constant curvature case gave such good results in a much simpler manner. These results should, therefore, indicate why the non-constant curvature procedure was abandoned in favor of the constant curvature technique.

2. Inverse Design of pxq Grillage

Having decided upon the constant curvature approach, the inverse design procedure for a p x q grillage was developed in Appendix A for simply supported and fully clamped ends. The

final result for simply supported ends are equations (26) and (27) which give the required variation of I_x^j and I_y^i respectively, and equation (28) which gives the appropriate values of c. For fully clamped ends, equations (53),(54),(55), and (56) give the required I's while (56a) determines the necessary c's.

It is obvious from the design equations that clamping the ends complicates the design procedure by increasing the number of ways in which the deflections must be matched. With this type of design procedure, any partial degree of end fixation must be treated separately, since the matching of deflections is dependent upon the location of the point of inflection. A possible solution for this drawback would be to precompute the design equations for several discrete values of end fixation. Then, by placing the bulk of the equations in the same form it might be possible to interpolate for the correct constant factors, depending upon the desired degree of fixation.

3. Weight Independent of R's

At first glance, the design equations for I might appear to be incomplete since they contain the redundant internal reactions R. However, the result obtained from Appendix C (2) is:

> For beams with rectangular (symmetric) cross sections, the total beam weight is independent of the values of the redundant reactions. A closed form of the weight equation

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for a grillage with beams of rectangular cross sections is then given by (67) and (68).

If the basic premise that σ = constant is the only criterion for minimum weight, then the above statement can be extended to beams with any cross sectional geometry by the following proof.

$$\sigma = \frac{Mc}{I} = \frac{M}{Z} = \text{constant}$$

then a) $I \sim M$ for constant curvature

and b) $Z \sim M$ for non-constant curvature

Since any design which results in σ = constant everywhere in the extreme fiber is a minimum weight design, both a) and b) above are minimum weight designs. From this we see that the minimum weight design is obtained by merely making I or Z proportional to the applied bending moment, which contains the redundant reactions R. As long as I or Z remains proportional to the applied moment, whatever it may be, constant stress will result and the weight will be a minimum. Hence, the weight of the structure is independent of the magnitude of the redundant reactions, regardless of the shape of the beam cross section. The only questionable aspect is whether or not such a design represents the absolute minimum weight attainable. While this has not been proven rigorously, the authors feel that the results of this thesis suggest that this is indeed the case. Whether or not the reader accepts this is solely his own judgment. However,

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it cannot be denied that the weight savings produced by this method are considerable and well worth a closer look.

4. Plated Tee Properties

Application of the inverse design procedure to unsymmetrical plated tee beams required detailed analysis of the geometrical properties of plated tees. In order to satisfy the requirement of constant c it was necessary to analyze stiffeners with wide variations in proportion. These stiffeners were placed on plates of constant thickness and width. The variations in stiffener proportions were related such that there were only three independent parameters; t, w_f, and r. The relations between the parameters are found in both Appendices A and B. Plate thickness was held constant for any one set of stiffeners with w_f and r varying over ranges which were judged to be practical limits. The resulting variations in values of c and I were calculated by utilizing the IBM 7090 computer at the M.I.T. Computation Center. Plotting the values of c versus I for the range of r and w_f revealed the practical range for which a tee stiffener on a given plate flange could be utilized. By constructing a line of constant c on the plot, the possible variation in I achieved by varying only flange width and tee depth could be found. It is also possible to

place lines of constant flange width on the c-I plots but by doing so it was found that confusion resulted. Therefore, separate curves giving the dimensional flange width for given values of I and r were constructed. Thus, by a two step graphical procedure it is possible to find the stiffener which satisfies the prescribed c and inertia (provided the inertia is somewhat greater than zero). Figures VII through XXII present the results of this investigation for plate thickness from 1/8 through 1 inch in 1/8 inch increments.

The use of the equations developed in Appendix A for the required inertia variations and values of c were used in conjunction with the c-I and r-I curves in a practical grillage design, Appendix B, part III. A 3 x 3 grillage was designed completely including examination of shear requirements at points where the desired inertia value approached zero. The design required total stiffener weight of 1886.8 lbs. The same grillage designed by conventional techniques, Appendix B, part IV, and using prismatic members was also made for comparison of stiffener weight. The total weight of stiffeners in this conventional design was 3480 lbs. The stiffeners designed by the proposed method thus were 54% of the weight of the stiffeners designed by conventional prismatic methods. Including the plating weight showed that the grillage designed by the proposed method has a

total weight of 7762 lbs. as opposed to 9355 lbs. for the conventional design, nearly a 20% weight saving.

For the sample 3 x 3 grillage the conventionally designed stiffeners were not of significantly greater weight for the lighter y beams. The weight saving was predominantly in the heavier beams. This would tend to indicate that the greatest proportion of the weight saving available by use of the proposed inverse technique can still be achieved by combining the proposed and conventional procedures. Any member which is much smaller than the heavier members of a grillage would apparently not carry any great weight penalty if conventionally designed. A practical means of determining the point at which conventional design for the light members is appropriate would appear to be by use of the c-I plot. If the required r values are found to be less than .6, the lowest value plotted, and the heavier members are in the practical range of the curve, then the light member could be made prismatic.

Figure XXVI shows the configuration of the central x beam in the grillage designed by the inverse method. The variation in depth over most of the length is not great. In fact it is very small in comparison to the total length (about 2 inches in the central 22 feet of the beam). This means that the required variation in inertia over this same 22 feet could very

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closely be achieved by simply varying the flange width and keeping the depth of the tee constant. The depth could then be reduced at the extreme ends to that required by shear considerations. It would not seem unreasonable, then, to select a standard rolled shape which satisfies the maximum I and c values (near the center of the span) and merely taper the flange so as to approach the required variation in inertia. This technique would still approximate the required constant curvature.

In general, the results of this thesis indicate that the proposed inverse design procedure can be both practical and advantageous. When stiffener dimensions become so small as to exceed the limits of the design charts, the stiffener can be made prismatic with only a very small weight penalty. As long as the heavy members are within the limits of the design curves, a very substantial weight saving can be achieved. Since plating weight is such a large proportion of the grillage weight, it would appear to be very desirable to keep the plate panel size as small as practical in order to reduce the plating weight. Even though the smaller panel size requires more stiffeners, the stiffener weight will be a minimum for the given grillage configuration by the inverse design procedure. It has also been shown that the variation in inertia

can be very closely approximated with a constant depth tee stiffener.

It is recommended that further work be done to correlate the maximum values of I and c as determined by this method with those obtained with standard rolled shapes. The final practical result would then be to use a standard rolled shape for the tee stiffener and trim the flange as necessary to get the required variation in inertia. This would permit a substantial saving in weight with only a small increase in the complexity of fabrication.

Although the authors discarded the non-constant curvature approach due to its mathematical complexity, additional work on this approach is recommended. Since non-constant curvature implies non-constant c as long as σ remains constant, a greater range of practical I values could be achieved from the varying tee stiffeners by controlling the c value.

IV CONCLUSIONS AND RECOMMENDATIONS

The results of this thesis indicate that the proposed design procedure is both practical and highly advantageous from the standpoint of minimizing structural weight. Another outstanding advantage, having nothing to do with minimum weight, is that the grillage can be designed to satisfy both deflection and stress requirements simultaneously in just one step. Thus, the trial and error procedure of designing indeterminate structures has been circumvented. As pointed out in Section III, the weight of the grillage designed for constant stress is independent of the magnitude of the internal reactions. Thus, they can be assigned any arbitrary values (even zero) by the designer, and in this way they can, to some extent, help control the shape of the resulting stiffeners. The highly indeterminate grillage, then, has been reduced to a relatively simple determinate structure by the proposed design technique.

A general conclusion regarding the use of the design curves is that if the required value of r is less than the lowest value plotted (0.6), the corresponding stiffener may be a prismatic member with very little weight penalty. The

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heavier the member, the greater the difference in weight between the constant stress beam and the conventional prismatic beam. As pointed out in Section III, the variation in beam depth is quite small compared to the length so that a feasible practical application would be to use a constant depth tee beam, and obtain the necessary variation in inertia by merely trimming the flange as necessary. While not precisely satisfying the requirement of constant c this would be a very close approximation, and much more practical than varying both flange and depth.

In order to make this design procedure even more practical, it is recommended that more work be done to correlate the maximum values of I and c obtained from this design procedure with those obtainable from standard rolled shapes attached to plating. It is envisioned that the final result would be a set of design curves which would enable the designer to select a standard rolled shape for the stiffener based on the maximum values of I and c required for the plate-stiffener combination. Then by merely cutting the stiffener flange the designer would be able to approximate closely the required inertia variation.

Since the plating weight is such a large part of the total

grillage weight, it would seem desirable to keep the plate panel size as small as practical in order to reduce the weight of the plating. Although this will require relatively more stiffeners, the stiffener weight can always be minimized for any particular configuration in order to keep the total weight as small as possible.

Although the authors abandoned the non-constant curvature approach due to its mathematical complexity, it is felt that further investigation should be done in this area. Using constant c limits the variation of I to a rather small range on the design charts. Quite possibly some other deflected shape can be found which will more fully utilize the scope of the design curves. Certainly, this possibility should not be completely ignored until it has been more carefully examined.

Only the extreme cases of simple supports and fully clamped ends were examined in this thesis. Most real structures, however, will probably fall somewhere between these two extremes. It would be very desirable to examine various other degrees of end fixation, and see if the design equations followed enough of a pattern to permit the designer to "interpolate" as necessary.

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Since the procedure developed here considered only a uniform pressure loading, the design technique should also be expanded to include point loads and possibly a non-uniform pressure loading. As with most grillage work, compressive end loads have not been considered. With the constant stress stiffeners produced by the proposed design method, the buckling problem due to end loads becomes more complicated since the beams are no longer prismatic members. This problem should be very thoroughly examined before any grillage designed by this method is subjected to a combination of lateral and end loads.

Although the stiffeners designed by the inverse method are of minimum weight, the minimum applies only to the particular type of stiffener being used. It is quite possible that a different stiffener geometry might be lighter than the tee beams considered. The tee stiffeners were studied because they are used to a very large extent in naval ship construction. Although the same design procedure could be used for other stiffener geometries, it is felt that the tee stiffener is best suited for this type of design technique.

The sections at the supports were designed on the basis of shear requirements although the possibility of web instability

in this region was not examined. Good design practice would normally require an investigation of web stability and corrective measures should be taken where needed. However, the weight involved in preventing web instability at the supports will almost always be quite small, and therefore not appreciably affect the minimum weight design.

The final point to note is that with this design procedure, the entire grillage is at the same level of stress. Consequently the structure is much closer to ultimate collapse than the corresponding grillage with prismatic members. In the minimum weight design, yielding will occur simultaneously in all parts of the grillage. Thus, the margin between yield and collapse is much smaller in the minimum weight design than in the conventional design. Therefore, when using this design procedure it may be necessary to use a larger factor of safety than that normally used with a conventional design. This increased factor of safety may reduce the weight saving obtained from the design procedure, and the magnitude of this effect should also be investigated.

V APPENDIX

A. Details of Procedure

B. Summary of Data and Calculations

C. Sample Calculations

D. Bibliography

A. DETAILS of PROCEDURE.

In this section, the techniques outlined in Section II will be applied to a flat, plated grillage, stiffened by p longitudinal beams and q transverse beams, as shown in Figure I. . The grillage is subjected to a uniform hydrostatic pressure P, and this pressure is distributed to the beams as line loads, as outlined below. For the case of constant curvature, both simply supported and fully clamped ends will be investigated. By considering the special case of beams with rectangular cross sections, a closed form of the weight equation can be obtained which sheds a very interesting light on the entire problem. In addition, a sixth order deflected shape will be investigated for use in the design procedure involving non-constant curvatures.

The extension of the above procedures to the plated tee beams making up the grillage in a ship's bottom will be outlined. Expressions for the moment of inertia and the depth to neutral axis of the general beam cross section will be developed. From these expressions it will be possible to construct curves relating c and I for large variations in stiffener proportions and for several plating thicknesses.

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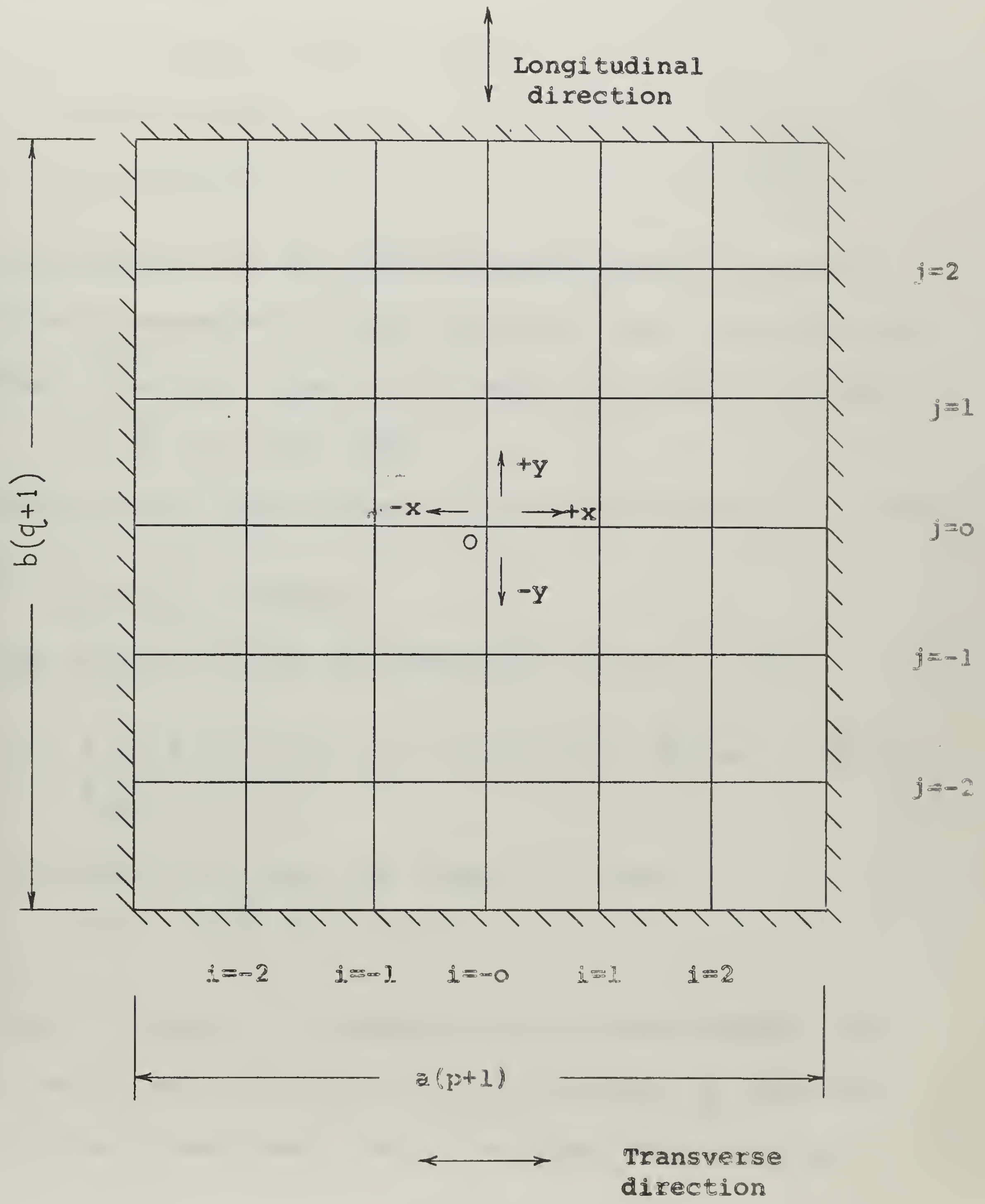
FIGURE I

Example of Nomenclature for p=5, q=5 Grillage



1. Determination of Qx and Qy from Pressure P

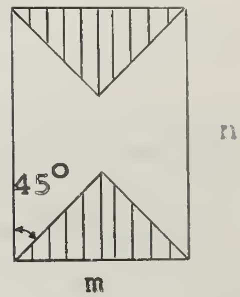
Consider any single plate panel in overall grillage.

P = Uniform pressure on panel

m = Short side

n = Long side

Figure II



It is assumed that the area enclosed by the 45° triangle effectively transmits its load to the "m" beam, the short side stiffener. Then F_m = Force on "m" beam = P x Area of Triangle

$$= P\ (\tfrac{1}{2})\ (m)\ (\frac{m}{2}) = \frac{P\,m^2}{4}$$

Distributing this total force uniformly over the length of "m", we get

$$q_m = \frac{F_m}{m} = \frac{P\,m}{4}$$

Then the remaining force is transmitted to the "n" beam.

$$F_n = P\ (\tfrac{1}{2}) \left[A\ (\text{total}) - 2A\ (\text{triangle}) \right] = \frac{P}{2}\left[mn - 2\ \frac{m^2}{4} \right]$$

$$F_n = \frac{P\,m}{4}\left[2\ n - m \right]$$

and distributing this over the length "n" gives

$$q_n = \frac{F_n}{n} = \frac{P\ m}{4}\left[2 - m/n \right]$$

Since each stiffener is a member of two adjacent panels, the total distributed load carried by the stiffener is twice the contribution of one panel. Then, the total beam loads are:

[illegible]

$$Q_S = 2\, q_m = \frac{P\, m}{2} = \text{Load on short beam} \qquad (4)$$

$$Q_L = 2\, q_n = \frac{P\, m}{2}\left[2 - {}^{m}/_{n}\right] = \text{Load on long beam} \qquad (5)$$

Equations (4) and (5) are to be used to determine Q_x and Q_y, using the appropriate equations for the long and short side.

2. Simply Supported, p x q Grillage, under Hydrostatic Load

Step 1: Assume the deflected shape

$$w = A\, x^{2} + B\, x + C \qquad (6)$$

Consider the central transverse beam

$$w_x^j = w_x^o = A\, x^2 + B\, x + C \qquad (7)$$

The boundary conditions are:

$$\text{At } x = o, \quad w_x^o = - w_o \text{ and } \frac{dw}{dx} = 0$$

$$\text{At } x = -\frac{a}{2}\,(p + 1), \quad v_x^o = 0$$

Applying the boundary condition to (7) we get

$$w_x^o = w_o \left[\frac{4x^2}{a^2 (p + 1)^2} - 1\right] \qquad (8)$$

For the side transverses, the expression is almost the same. The difference is that at $x = o$, $w_x^j = w_y^o$ in order for all deflections at beam intersections to be compatible.

Thus, with this change we get

$$w_x^j = w_y^o \left[1 - \frac{4x^2}{a^2(p+1)^2} \right] \tag{9}$$

A similar process is followed for the deflections of the longitudinal beams, with the following results:

$$w_y^o = w_o \left[\frac{4y^2}{b^2(q+1)^2} - 1 \right] \tag{10}$$

$$w_y^i = w_x^o \left[1 - \frac{4y^2}{b^2(q+1)^2} \right] \tag{11}$$

Combining (8), (9), (10), and (11) we arrive at the generalized deflection expressions for any longitudinal or transverse beam.

$$w_x^j = w_o \left[\frac{4y^2}{b^2(q+1)^2} - 1 \right] \left[1 - \frac{4\ x^2}{a^2(p+1)^2} \right] \tag{12}$$

$$w_y^i = w_o \left[\frac{4x^2}{a^2(p+1)^2} - 1 \right] \left[1 - \frac{4y^2}{b^2(q+1)^2} \right] \tag{13}$$

From these expressions we now determine the curvatures in the x and y directions.

x Direction: $$w_x^{j''} = \frac{d^2 w_x}{d\ x^2} = w_o \left[\frac{4y^2}{b^2(q+1)^2} - 1 \right] \left[\frac{-8}{a^2(p+1)^2} \right] \tag{15}$$

y Direction: $$w_y^{i''} = \frac{d^2 w_y}{d\ y^2} = w_o \left[\frac{4x^2}{a^2(p+1)^2} -1 \right] \left[\frac{-8}{b^2(q+1)^2} \right] \tag{16}$$

Since the grillage consists of discrete beams, the curvature for any particular transverse is determined by using the value of y appropriate to the particular transverse in the expression for w_x'' and similarly for w_y'' Since the x and y position of the longi-

tudinals and transverses, respectively, is an integral number of beam spacings from the origin, we can replace x by ia and y by jb. With this substitution then, the curvatures become:

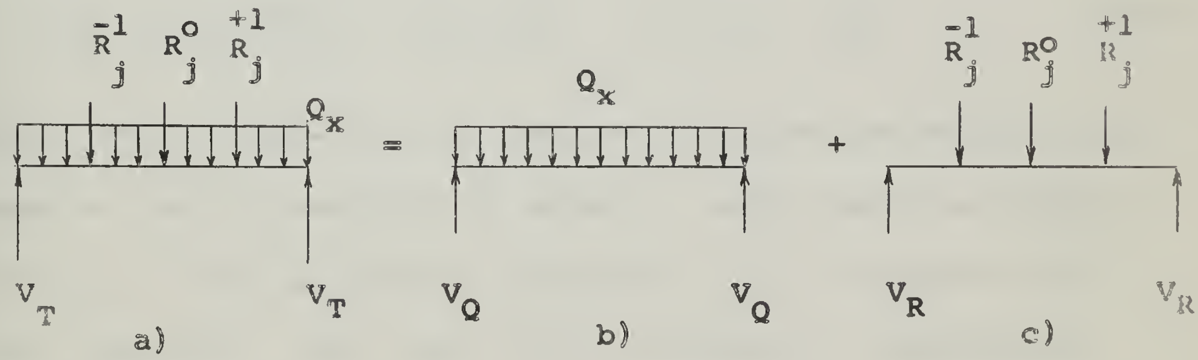
$$w_x^{j\,''} = w_o \left[\frac{4\, j^2}{(q+1)^2} - 1\right] \left[\frac{-8}{a^2 (p+1)^2}\right] \qquad (17)$$

$$w_y^{i\,''} = w_o \left[\frac{4\, i^2}{(p+1)^2} - 1\right] \left[\frac{-8}{b^2\ (q+1)^2}\right] \qquad (18)$$

Step 2: From equilibrium conditions, determine the expression for the bending moment along a stiffener.

To get the moment expression, the principle of superposition will be used. Consider first the transverse beams. With superposition the loading can be considered as shown in the following diagram.

FIGURE III



For the distributed load

$$M_x^j = \frac{XQ_x}{2}\ (L - X) \qquad (19)$$

when X is measured from the left hand edge of the beam. With the origin at the center of the beam $X = x + \frac{a}{2}\ (p + 1)$

and

$$L = a \quad (p+1)$$

Substituting these into (19) we get:

$$M_x^j = Q_x \left[\frac{a^2(p+1)^2}{8} - \frac{x^2}{2} \right] \tag{20}$$

For the concentrated loads we will utilize singularity notation [3]. Referring to Figure III C)

$$V_r = \tfrac{1}{2} \sum_i R_j^i = \tfrac{1}{2} \sum_{i=-\frac{(p-1)}{2}}^{i=\frac{(p-1)}{2}} R_j^i$$

Starting from the left hand end of the beam:

$$M_x^j = \tfrac{1}{2} \sum_i R_j^i \left[x + \frac{a}{2}(p+1) \right] - R_j^{-1} \left\langle x + \frac{a}{2}(p+1) - a \right\rangle \tag{21}$$

$$- R_j^o \langle x \rangle - R_j^1 \langle x - a \rangle$$

or writing the general form in terms of the integral index i, we get:

$$M_x^j = \tfrac{1}{2} \sum_i R_j^i \left[x + \frac{a}{2}(p+1) \right] - \sum_i R_j^i \langle x - ia \rangle \tag{22}$$

where the $\langle\ \rangle$ term is to be ignored if the value within $\langle\ \rangle$ is negative, and $\langle\ \rangle$ replaced by normal ( ) when the term becomes positive. Once the term is positive, it is retained in the basic equation. Note that (22) is written for the moments starting at the left hand end of the beam where both x and i are negative.

Now, to get the total general moment expression for any transverse beam, simply add (20) and (22). This gives:

$$M_x^j = Q_x \left[\frac{a^2(p+1)^2}{8} - \frac{x^2}{2}\right] + \tfrac{1}{2}\sum_i R_j^i \left[x + \frac{a}{2}(p+1)\right] - \sum_i R_j^i \langle x - ia \rangle \quad (23)$$

The same procedure applies to the longitudinal beams, and thus by direct analogy we have:

$$M_y^i = Q_y \left[\frac{b^2(q+1)^2}{8} - \frac{y^2}{2}\right] + \tfrac{1}{2}\sum_j R_i^j \left[y + \frac{b}{2}(q+1)\right] - \sum_j R_i^j \langle y - jb \rangle \quad (24)$$

<u>Step 3:</u> <u>Relate Bending Moments to Curvatures</u>.

Again, consider the transverse beams first. From equation (1) we see that

$$E\, I_x^j\, w_T^{j\,''} = M_x^j$$

and, making the appropriate substitutions from (17) and (23) we get:

$$E\, I_x^j\, w_0 \left[\frac{4_j^2}{(q+1)^2} - 1\right] \left[\frac{-8}{a^2(p+1)^2}\right] = Q_x \left[\frac{a^2(p+1)^2}{8} - \frac{x^2}{2}\right]$$
$$+ \tfrac{1}{2} \sum_i R_j^i \left[x + \frac{a}{2}(p+1)\right] - \sum_i R_j^i \langle x - ia \rangle \quad (25)$$

Solving for I_x^j gives:

$$I_x^j = -\frac{a^2(p+1)^2}{8E\, w_0} \left[\frac{(q+1)^2}{4_j^2 - (q+1)^2}\right] \left\{ Q_x \left[\frac{a^2(p+1)^2}{8} - \frac{x^2}{2}\right] + \tfrac{1}{2} \sum_i R_j^i \right.$$
$$\left. \left[x + \frac{a}{2}(p+1)\right] - \sum_i R_j^i \langle x - ia \rangle \right\} \quad (26)$$

Again, by direct analogy, the expression for the longitudinal beams is:

$$I_y^i = -\frac{b^2(q+1)^2}{8\, EW_0} \left[\frac{(p+1)^2}{4_j^2 - (p+1)^2}\right] \left\{ Q_y \left[\frac{b^2(q+1)^2}{8} - \frac{y^2}{2}\right] \right.$$
$$\left. + \tfrac{1}{2} \sum_j R_i^j \left[y + \frac{b}{2}(q+1)\right] - \sum_j R_i^j \langle y - jb \rangle \right\} \quad (27)$$

Step 4: Relate Stress to Curvature

$$\sigma = E\ c\ w'' = \text{Constant}$$

Thus

$$\sigma = E\ c_x^j\ w_x^{j''} = E\ c_y^i\ w_y^{i''}$$

and for this case using equations (17) and (18) we get:

$$\frac{\sigma}{E\ w_o} = c_x^j \left[\frac{4j^2}{(q+1)^2} - 1 \right] \left[\frac{-8}{a^2(p+1)^2} \right] = c_y^i \left[\frac{4i^2}{(p+1)^2} - 1 \right] \left[\frac{-8}{b^2(q+1)^2} \right] \quad (28)$$

3. p x q Grillage under Hydrostatic Load with Clamped Edge Conditions

Step 1: Again assume a second order deflected shape.

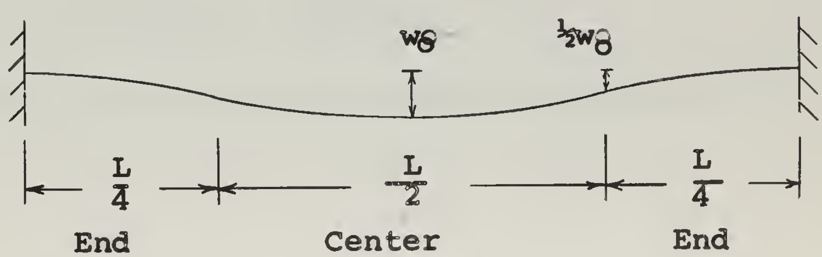
For $\sigma = E\ c\ w''$

and $c = \text{Constant}$

$w'' = \text{Constant}$ along entire length.

Thus, by geometric consideration, the deflected shape must consist of arcs of equal, constant curvature, with inflection points at the quarter points of the total span. The deflected profile must then be as shown below.

FIGURE IV



Then, the deflected shape in each section, center and end, will be of the form

$$w = A x^2 + Bx + C \tag{6}$$

Considering the central transverse#, the boundary conditions for the central section are:

At $x=0$, $w_c^{OT} = -w_o$ and $\frac{dw}{dx} = 0$

At $x = \frac{a}{4}(p+1)$, $w_c^{OT} = -\frac{w_o}{2}$

Substituting these into (6) gives:

$$w_c^{OT} = w_o \left[\frac{8x^2}{a^2(p+1)^2} - 1 \right] \tag{29}$$

For the end section the boundary conditions are:

At $x = \frac{a}{2}(p+1)$, $w_e^{OT} = 0$ and $\frac{dw}{dx} = 0$

At $x = \frac{a}{4}(p+1)$, $w_e^{OT} = -\frac{w_o}{2}$

Substituting these into (6) gives:

$$w_e^{OT} = -w_o \left[\frac{8x^2}{a^2(p+1)^2} - \frac{8x}{a(p+1)} + 2 \right] \tag{30}$$

The deflection equations for the central longitudinals can be written by analogy.

$$w_c^{OL} = w_o \left[\frac{8y^2}{b^2(q+1)^2} - 1 \right] \tag{31}$$

$$w_e^{OL} = -w_o \left[\frac{8y^2}{b^2(q+1)^2} - \frac{8y}{b\,(q+1)} + 2 \right] \tag{32}$$

For the side transverses the boundary conditions are:

At $x = 0$, $w_c^{jT} = w_c^{OL}$ or w_e^{OL} and $\frac{dw}{dx} = 0$

At $x = \frac{a}{4}(p+1)$, $w_c^{jT} = \frac{1}{2} w_c^{OL}$ or $\frac{1}{2} w_e^{OL}$

The point at which the side transverse intersects the central longitudinal along its length will determine whether w_c^{OL} or w_e^{OL} should be used. The deflection expression changes at the quarter point so if $|y| \leq \frac{b}{4}(q+1)$ use w_c^{OL} and if $|y| > \frac{b}{4}(q+1)$, use w_e^{OL}. With these restrictions, and the boundary conditions we get:

$$w_c^{jT} = \left[1 - \frac{8x^2}{a^2(p+1)^2}\right] w_c^{OL} \quad \text{for } -\frac{b}{4}(q+1) \leq y \leq \frac{b}{4}(q+1)$$

or $$w_c^{jT} = \left[1 - \frac{8x^2}{a^2(p+1)^2}\right] w_e^{OL} \quad \text{for } |y| > \frac{b}{4}(q+1)$$

Substituting for w_c^{OL} and w_e^{OL} and noting that for the discrete beams $y = jb$ for any particular beam, we get:

$$w_c^{jT} = w_o \left[1 - \frac{8x^2}{a^2(p+1)^2}\right]\left[\frac{8j^2}{(q+1)^2} - 1\right] \quad \text{for } |j| \leq \frac{q+1}{4} \tag{33}$$

$$w_c^{jT} = w_o \left[\frac{8x^2}{a^2(p+1)^2} - 1\right]\left[\frac{8j^2}{(q+1)^2} - \frac{8j}{(q+1)} + 2\right] \quad \text{for } |j| > \frac{q+1}{4} \tag{34}$$

For the end sections of the side transverses, the boundary conditions are:

At $x = \frac{a}{2}(p+1)$, $w_e^{jT} = 0$ and $\frac{dw}{dx} = 0$

At $x = \frac{a}{4}(p+1)$, $w_e^{jT} = \frac{1}{2} w_c^{OL}$ or $\frac{1}{2} w_e^{OL}$

Carrying out the same procedure as before, we get:

$$w_e^{jT} = w_o \left[\frac{8x^2}{a^2(p+1)^2} - \frac{8x}{a(p+1)} + 2\right]\left[\frac{8j^2}{(q+1)^2} - 1\right] \quad \text{for } |j| \leq \frac{q+1}{4} \tag{35}$$

$$w_e^{jT} = -w_o \left[\frac{8x^2}{a^2(p+1)^2} - \frac{8x}{a(p+1)} + 2\right]\left[\frac{8j^2}{(q+1)^2} - \frac{8i^2}{(q+1)} + 2\right]$$

$$\text{for } |j| > \frac{q+1}{4} \tag{36}$$

Now by analogy we can immediately write the deflection expressions for the longitudinals.

$$w_c^{iL} = w_o \left[1 - \frac{8y^2}{b^2(q+1)^2}\right]\left[\frac{8i^2}{(p+1)^2} - 1\right] \quad \text{for } |i| \leq \frac{p+1}{4} \tag{37}$$

$$w_c^{iL} = w_o \left[\frac{8y^2}{b^2(q+1)^2} - 1\right]\left[\frac{8i^2}{(p+1)^2} - \frac{8i}{(p+1)} + 2\right] \text{for } |i| > \frac{p+1}{4} \tag{38}$$

$$w_e^{iL} = w_o \left[\frac{8y^2}{b^2(q+1)^2} - \frac{8y}{b(q+1)} + 2\right]\left[\frac{8i^2}{(p+1)^2} - 1\right] \quad \text{for } |i| \leq \frac{p+1}{4} \tag{39}$$

$$w_e^{iL} = -w_o \left[\frac{8y^2}{b^2(q+1)^2} - \frac{8y}{b(q+1)} + 2\right]\left[\frac{8i^2}{(p+1)^2} - \frac{8i}{(p+1)} + 2\right]$$

$$\text{for } |i| > \frac{p+1}{4} \tag{40}$$

Now, differentiate twice to get the curvatures.

<u>X Direction</u>

Center: $$\frac{d^2}{dx^2}\left(w_c^{jT}\right) = -\frac{16\, w_o}{a^2(p+1)^2}\left[\frac{8j^2}{(q+1)^2} - 1\right] \quad |j| \leq \frac{q+1}{4} \tag{41}$$

$$\frac{d^2}{dx^2}\left(w_c^{jT}\right) = \frac{16\, w_o}{a^2(p+1)^2}\left[\frac{8j^2}{(q+1)^2} - \frac{8j}{(q+1)} + 2\right] \quad |j| > \frac{q+1}{4} \tag{42}$$

Ends: $$\frac{d^2}{dx^2}\left(w_e^{jT}\right) = \frac{16\ w_o}{a^2(p+1)^2}\left[\frac{8j^2}{(q+1)^2} - 1\right] \quad |j| \leq \frac{q+1}{4} \tag{43}$$

$$\frac{d^2}{dx^2}\left(w_e^{jT}\right) = -\frac{16\ w_o}{a^2(p+1)^2}\left[\frac{8j^2}{(q+1)^2} - \frac{8j}{(q+1)} + 2\right] \quad |j| > \frac{q+1}{4} \tag{44}$$

$\underline{y}$ Direction

Center: $$\frac{d^2}{dy^2}\left(w_c^{iL}\right) = -\frac{16\ w_o}{b^2(q+1)^2}\left[\frac{8i^2}{(p+1)^2} - 1\right] \quad |i| \leq \frac{p+1}{4} \tag{45}$$

$$\frac{d^2}{dy^2}\left(w_c^{iL}\right) = \frac{16\ w_o}{b^2(q+1)^2}\left[\frac{8i^2}{(p+1)^2} - \frac{8i}{(p+1)} + 2\right] \quad |i| > \frac{p+1}{4} \tag{46}$$

Ends: $$\frac{d^2}{dy^2}\left(w_e^{iL}\right) = \frac{16\ w_o}{b^2(q+1)^2}\left[\frac{8i^2}{(p+1)^2} - 1\right] \quad |i| \leq \frac{p+1}{4} \tag{47}$$

$$\frac{d^2}{dy^2}\left(w_e^{iL}\right) = \frac{-16\ w_o}{b^2(q+1)^2}\left[\frac{8i^2}{(p+1)^2} - \frac{8i}{(p+1)} + 2\right] \quad |i| > \frac{p+1}{4} \tag{48}$$

Step 2: Express the bending moment in terms of the load, reactions, and end moment.

The moment equations will be the same as before, except that a third term, the end moment, must be added.

Thus,

$$M_x^j = Q_x\left[\frac{a^2(p+1)^2}{8} - \frac{x^2}{2}\right] + 1/2 \sum_i R_j^i\left[x + \frac{a}{2}(p+1)\right]$$
$$- \sum_i R_j^i \langle x - ia\rangle - M_c^j \tag{49}$$

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$$M_y^i = Q_y\left[\frac{b^2(q+1)^2}{8} - \frac{y^2}{2}\right] + \tfrac{1}{2}\sum_j R_i^j\left[y + \frac{b}{2}(q+1)\right]$$
$$- \sum_j R_i^j \langle y - jb\rangle - M_c^i \qquad (50)$$

<u>Step 3</u>: Relate the bending moments to the curvatures.

For the transverses, when $|j| \leqslant \frac{q+1}{4}$

Center: $$EI_x^j \left[\frac{-16w_o}{a^2(p+1)^2}\right]\left[\frac{8j^2}{(q+1)^2} -1\right] = M_x^j \qquad (51)$$

Ends: $$EI_x^j \left[\frac{16\ w_o}{a^2(p+1)^2}\right]\left[\frac{8j^2}{(q+1)^2} -1\right] = M_x^j \qquad (52)$$

where M_x^j is given by equation (49). However, since we are interested in the variation of I_x, the change in sign of the curvature (moment) merely reverses the sign of the stress, but does not affect its magnitude, and thus does not affect the magnitude of the variation of I_x. Thus, the change of sign of the curvature can be ignored when determining I_x. Then, for the total length of the beam,

$$I_x^j = -\frac{a^2(p+1)^2}{16\ E\ w_o}\left[\frac{(q+1)^2}{8j^2-(q+1)^2}\right] M_x^j\ , \quad |j| \leq \frac{(q+1)}{4} \qquad (53)$$

The same reasoning applies to all the other beams, so

$$I_x^j = \frac{a^2(p+1)^2}{16\ E\ w_o}\left[\frac{(q+1)^2}{8j^2-8j(q+1)+2(q+1)^2}\right] M_x^j, |j| > \frac{q+1}{4} \qquad (54)$$

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and similarly, in the y direction.

$$I_y^i = - \frac{b^2(q+1)^2}{16\ E\ w_o} \left[\frac{(p+1)^2}{8i^2-(p+1)^2}\right]^2 M_y^i, \ |i| \leq \frac{p+1}{4} \qquad (55)$$

$$I_y^i = \frac{b^2(q+1)^2}{16\ E\ w_o} \left[\frac{(p+1)^2}{8i^2-8i(p+1)+2(p+1)^2}\right]^2 M_y^i, \ |i| > \frac{p+1}{4} \qquad (56)$$

where M_y^i is given by equation (50).

Step 4: Relate stress to curvature.

Following the same procedure as in the case of simple supports:

$$\sigma = E\ c_x^j\ w_x^{j\,''} = E\ c_y^i\ w_y^{i\,''} = \text{Constant} \qquad (56a)$$

Using the appropriate curvature for each particular beam, the proper value of c for that beam is obtained from the above equation.

4. Grillage Weight for Beams of Rectangular Cross Section.

In general, the total weight of the grillage will be the sum of the weights of the longitudinal and transverse beams. Thus,

$$W = W\ (\text{long.}) + W\ (\text{trans.})$$

considering just a gridwork of beams with no plating. But

$$W\ (\text{long.}) = \sum_i 2\gamma \int_o^{L/2} A_y^i\ dy \qquad (57)$$

and

$$W\ (\text{trans.}) = \sum_j 2\gamma \int_o^{L/2} A_x^j\ dx \qquad (58)$$

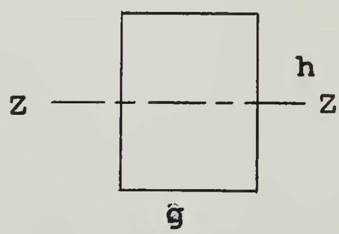
where

δ = Specific weight of material and by symmetry, the total weight is merely twice the weight of one half of the beam.

Consider now for simplicity, beams with rectangular cross sections. Then

$$I_{zz} = \frac{gh^3}{12}$$

Figure V



$$A = gh$$

so

$$A = \left(\frac{12}{h^2}\right) I_{zz}$$

and h is constant for any particular beam when the constant curvature design procedure is being used. Now, using equations (26) and (27) for a simply supported grid we get -

$$A_x^j = - \frac{12\ a^2(p+1)^2}{8\ E\ w_o (h_x^j)^2} \left[\frac{(q+1)^2}{4j^2-(q+1)^2}\right] \left\{ Q_x \left[\frac{a^2(p+1)^2}{8} - \frac{x^2}{2}\right] + 1/2 \sum_i R_j^i \left[x + \frac{a}{2}(p+1)\right] - \sum_i R_j^i \langle x - ia \rangle \right\} \quad (59)$$

$$A_y^i = - \frac{12\ b^2(q+1)^2}{8\ E\ w_o (h_y^i)^2} \left[\frac{(p+1)^2}{4i^2-(p+1)^2}\right] \left\{ Q_y \left[\frac{b^2(q+1)^2}{8} - \frac{y^2}{2}\right] + 1/2 \sum_j R_i^j \left[y + \frac{b}{2}(q+1)\right] - \sum_j R_i^j \langle y - jb \rangle \right\} \quad (60)$$

Now, relate stress to curvature -

$$\sigma = E\ c\ w''$$

From (17), for the transverses we have -

[illegible]

$$w_x^{j\,''} = w_o \left[\frac{4j^2}{(q+1)^2} - 1\right]\left[\frac{-8}{a^2(p+1)^2}\right]$$

and $c_x^j = \frac{1}{2} h_x^j$

so that

$$\sigma = E\, w_o \, \tfrac{1}{2} h_x^j \left[\frac{4j^2}{(q+1)^2} - 1\right]\left[\frac{-8}{a^2(p+1)^2}\right]$$

The same procedure may be applied to the longitudinals, so that finally we have

$$\frac{\sigma}{E\, w_o} = \tfrac{1}{2} h_x^j \left[\frac{4j^2}{(q+1)^2} - 1\right]\left[\frac{-8}{a^2(p+1)^2}\right] \tag{61}$$

$$= \tfrac{1}{2} h_y^i \left[\frac{4i^2}{(p+1)^2} - 1\right]\left[\frac{-8}{b^2(q+1)^2}\right]$$

Substituting for $(h_x^j)^2$ and $(h_y^i)^2$ in (59) and (60) gives

$$A_x^j = -\frac{24\, E\, w_o}{\sigma^2 a^2 (p+1)^2}\,2\left[\frac{4j^2}{(q+1)^2} - 1\right]\left\{Q_x\left[\frac{a^2(p+1)^2}{8} - \frac{x^2}{2}\right] + \tfrac{1}{2}\sum_i R_j^i\left[x + \frac{a}{2}(p+1)\right] - \sum_i R_j^i \langle x - ia\rangle\right\} \tag{62}$$

$$A_y^i = -\frac{24\, E\, w_o}{\sigma^2 b^2 (q+1)^2}\,2\left[\frac{4i^2}{(p+1)^2} - 1\right]\left\{Q_y\left[\frac{b^2(q+1)^2}{8} - \frac{y^2}{2}\right] + \tfrac{1}{2}\sum_j R_i^j\left[y + \frac{b}{2}(q+1)\right] - \sum_j R_i^j \langle y - jb\rangle\right\} \tag{63}$$

Then, by substituting these into (57) and (58) and integrating from the left hand end of the beam to the center we get

$$W_x = \sum_j 2\gamma \int_{-\frac{a}{2}(p+1)}^{0} \left[-\frac{24\, E\, w_o}{\sigma^2 a^2 (p+1)^2}2\right]\left[\frac{4j^2}{(q+1)^2} - 1\right]\left\{Q_x\left[\frac{a^2(p+1)^2}{8} - \frac{x^2}{2}\right] + \tfrac{1}{2}\sum_i R_j^i\left[x + \frac{a}{2}(p+1)\right] - \sum_i R_j^i \langle x - ia\rangle\right\}\Bigg]dx$$

which integrates to

$$W_T = \sum_j - \frac{48\,\gamma\,E\,w_\Omega}{\sigma^2 (q+1)^2} \left[4j^2 - (q+1)^2 \right] \left\{ \frac{Q_x \; a\,(p+1)}{24} \right. \tag{64}$$

$$\left. + 1/16 \sum_i R^i_j - \frac{1}{2(p+1)^2} \sum_i R^i_j \langle - i \rangle^2 \right\}$$

Similarly, for the longitudinals -

$$W_L = \sum_i - \frac{48\,\gamma\,E\,w_\Omega}{\sigma^2 (p+1)^2} \left[4\,i^2 - (p+1)^2 \right] \left\{ \frac{Q_y \; b\,(q+1)}{24} \right.$$

$$\left. + 1/16 \sum_j R^j_i - \frac{1}{2(q+1)^2} \sum_j R^j_i \langle - j \rangle^2 \right\} \tag{65}$$

Note that the integration is carried out in the range where i and j are negative. Hence, the singularity term is always positive. The total weight then becomes -

$$W = - \frac{48\,\gamma\,E\,w_\Omega}{\sigma^2} \left\{ \sum_j \left[\frac{4j^2}{(q+1)^2} - 1 \right] \left[\frac{Q_x \; a\,(p+1)}{24} \right. \right.$$

$$\left. + 1/16 \sum_i R^i_j - \frac{1}{2(p+1)^2} \sum_i R^i_j \langle - i \rangle^2 \right]$$

$$+ \sum_i \left[\frac{4i^2}{(p+1)^2} - 1 \right] \left[\frac{Q_y \; b\,(q+1)}{24} + 1/16 \sum_j R^j_i \right.$$

$$\left. \left. - \frac{1}{2\,(q+1)^2} \sum_j R^j_i \langle - j \rangle^2 \right] \right\} \tag{66}$$

In Appendix C it is shown that the terms involving the internal reactions R^j_i and R^i_j cancel out from the weight equation. Consequently the total grillage weight for beams of rectangular cross section becomes

$$W = -\frac{2\,\gamma\,E\,w_o}{\sigma^2}\left\{\frac{a(p+1)}{(q+1)^2}\,Q_x\sum_j\left[4j^2-(q+1)^2\right] + \frac{b(q+1)}{(p+1)^2}\,Q_y\sum_i\left[4\,i^2-(p+1)^2\right]\right\} \tag{67}$$

with simply supported ends. Following exactly the same procedure for clamped end conditions leads to a similar equation.

$$W = \frac{96\,\gamma\,E\,w_o}{\sigma^2}\left[\frac{a\,Q_x\,(p+1)}{24} - \frac{a}{2}\,M_c^j\,(p+1)\right]\left\{\sum_{j=-\frac{(q+1)}{4}}^{j=\frac{q+1}{4}}\left[1-\frac{8j^2}{(q+1)^2}\right] + 2\sum_{|j|>\frac{(q+1)}{4}}^{|j|=\frac{(q-1)}{2}}\left[\frac{8j^2}{(q+1)^2}-\frac{8j}{(q+1)}+2\right]\right\}$$

$$+\frac{96\,E\,w_o\,\gamma}{\sigma^2}\left[\frac{b\,Q_y\,(q+1)}{24} - \frac{b}{2}\,M_c^i\,(q+1)\right]\left\{\sum_{i=-\frac{p+1}{4}}^{i=\frac{p+1}{4}}\left[1-\frac{8i^2}{(p+1)^2}\right] + 2\sum_{|i|>\frac{p+1}{4}}^{|i|=\frac{p-1}{2}}\left[\frac{8i^2}{(p+1)^2}-\frac{8i}{(p+1)}+2\right]\right\} \tag{68}$$

5. Simply Supported p x q Grillage under uniform pressure with c and w" variable.

Step 1: Assume a deflected shape.

In order for c to be finite at x = o, it is necessary that any expression for w contain a second order term. The other higher order terms can be chosen as necessary to give a reasonable variation in w" and c. A relatively simple sixth order deflected shape is:

$$W = - W_o \left[1 - \left(\frac{2x}{L}\right)^2 - \tfrac{1}{2}\left(\frac{2x}{L}\right)^4 + \tfrac{1}{2}\left(\frac{2x}{L}\right)^6 \right] \tag{69}$$

For the central transverse and longitudinal, the expression becomes

$$W_x^o = - W_o \left[1 - \frac{4x^2}{a^2 (p+1)^2} - \frac{8\,x^4}{a^4 (p+1)^4} + \frac{32\,x^6}{a^6\,(p+1)^6} \right]$$

$$W_y^o = - W_o \left[1 - \frac{4y^2}{b^2 (q+1)^2} - \frac{8y^4}{b^4 (q+1)^4} + \frac{32\,y^6}{b^6 (q+1)^6} \right]$$

For the side beams, the expressions are:

$$W_x^j = W_y^o \left[1 - \frac{4\,x^2}{a^2 (p+1)^2} - \frac{8\,x^4}{a^4 (p+1)^4} + \frac{32\,x^6}{a^6 (p+1)^6} \right]$$

$$W_y^i = W_x^o \left[1 - \frac{4\,y^2}{b^2 (q+1)^2} - \frac{8\,y^4}{b^4 (q+1)^4} + \frac{32\,y^6}{b^6 (q+1)^6} \right]$$

Then, the curvatures are :

$$W_x^{j\,"} = W_y^o \left[- \frac{8}{a^2 (p+1)^2} - \frac{96\,x^2}{a^4 (p+1)^4} + \frac{960\,x^4}{a^6 (p+1)^6} \right] \tag{70a}$$

$$W_y^{i\,"} = W_x^o \left[- \frac{8}{b^2 (q+1)^2} - \frac{96\,y^2}{b^4 (q+1)^4} + \frac{960\,y^4}{b^6 (q+1)^6} \right] \tag{70b}$$

Step 2: Determine the moment expression.

The moment expressions have been previously developed and are given by (23) and (24), and they are independent of the deflected shape of the grillage.

Step 3: Relate the bending moments to the curvatures.

$$E\, I_x^j\, w_x^{j''} = M_x^j \quad \text{and} \quad E\, I_y^i\, w_y^{i''} = M_y^i$$

Thus

$$I_x^j = \frac{M_x^j}{E\, w_x^{j''}} \quad \text{and} \quad I_y^i = \frac{M_y^i}{E\, w_y^{i''}}$$

where $w_x^{j''}$ and $w_y^{i''}$ are given by (70a)and (70b).

Step 4: Relate stress to curvature to determine c.

$$\sigma = E\, c\, w''$$

so

$$C_x^j = \frac{\sigma}{E\, w_x^{j''}} \quad \text{and} \quad C_y^i = \frac{\sigma}{E\, w_y^{i''}}$$

In this case, the variation of the curvature prescribes the necessary variation of c.

6. Determination of geometrical properties of a tee beam attached to plating.

For an arbitrary cross section it is possible to find the centroid of area and, hence, the neutral axis and moment of inertia about that neutral axis by breaking the section up into units

[illegible]

[illegible]

[illegible]

[illegible]

$$[illegible]$$

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

of convenient geometry. For sections with one axis of symmetry the necessary calculations may be placed in a tabular form [9]. The neutral axis is easily found and then used in conjunction with the parallel axis theorem to find the moment of inertia about the neutral axis.

FIGURE VI

Nomenclature for Plated Tees

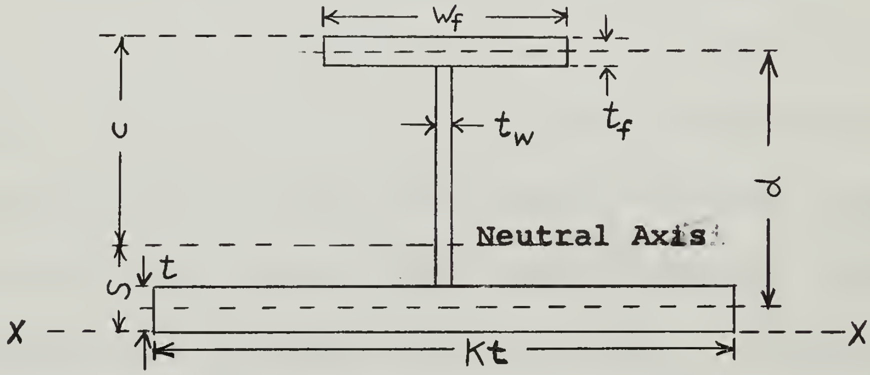


	Plate	Flange	Web
Area ($in.^2$)	kt^2	$w_f t_f$	$(d-\frac{t_f}{2}-\frac{t}{2})t_w$
Arm from XX(in)	$t/2$	$d+t/2$	$\frac{1}{2}(d-\frac{t_f}{2}-\frac{t}{2})+t$
First moment about XX(in^3)	$\frac{1}{2}kt^3$	$(d+\frac{t}{2})w_f t_f$	$(d-\frac{t_f}{2}-\frac{t}{2})t_w\left[\frac{1}{2}(d-\frac{t_f}{2}-\frac{t}{2})+t\right]$
I', second moment about XX(in)	$\frac{1}{4}kt^4$	$(d+\frac{t}{2})^2 w_f t_f$	$(d-\frac{t_f}{2}-\frac{t}{2})t_w\left[\frac{1}{2}(d-\frac{t_f}{2}-\frac{t}{2})+t\right]^2$
Icg(in^4)	$\frac{1}{12}kt^4$	$\frac{1}{12}w_f t_f^3$	$\frac{1}{12}t_w(d-\frac{t_f}{2}-\frac{t}{2})^3$

Using the expressions in the above table, the distance from XX to the neutral axis can be found since $S = \frac{\sum \text{First Moments}}{\sum \text{Areas}}$ (71)

The value of c, then, becomes:

$$c = d + t_f/2 + t/2 - S \qquad (72)$$

The moment of inertia of the section about the neutral axis is simply the sum of the second moment of each part about XX plus the sum of the moments about the individual centroids, corrected by the parallel axis theorem.

$$I = \sum I' + \sum Icg - \sum A\,(\delta)^2 \qquad (73)$$

The commonly used effective breadth of plating for mild steel of 60t [5] was selected to determine the amount of effective plating. A survey of [5] and [6] revealed that for plated tee sections the web thickness for typical sections in ship construction was about 60% of the flange thickness. The variation was seldom more than 10% from this figure. Furthermore, [5] and [6] indicated that an approximation to flange thickness of

$$t_f = 1/4(t + 1) \qquad (74)$$

would give reasonable values for plate thicknesses of one inch or less. To achieve a variety of stiffeners various ratios of flange width to tee depth were considered. Collectively these assumptions are listed below.

$$\text{Plate width} = 60t$$

$$t_f = 1/4\,(t+1) \qquad (74)$$

$$t_w = .6\,t_f \qquad (74a)$$

$$d = r\,w_f \qquad (74b)$$

The equations for determining c and I for a wide variation

in stiffener scantlings can then be written in a form easily adapted to machine computation.

$$S = \left\{ 30t^3 + w_f t_f (rw_f + \tfrac{1}{2}t) + t_w (rw_f - \tfrac{1}{2}t_f - \tfrac{1}{2}t)(\tfrac{1}{2}rw_f - \tfrac{1}{4}t_f + {}^3/_4 t) \right\}$$

$$\div \left\{ 60t^2 + w_f\, t_f + t_w\, (r\, w_f - \tfrac{1}{2}t_f - \tfrac{1}{2}t) \right\} \qquad (75)$$

$$C = rw_f + \tfrac{1}{2}\, t_f + {}^t/_2 - S \qquad (76)$$

$$I = 20t^4 + \left[{}^1/_{12} w_f t_f^{\,3} + w_f t_f (rw_f + \tfrac{1}{2}t)^2 \right] + \left\{ {}^1/_{12} t_w (rw_f - \tfrac{1}{2}t_f - \tfrac{1}{2}t)^3 \right.$$

$$+ t_w (rw_f - \tfrac{1}{2}\, t_f - \tfrac{1}{2}t) \quad (\tfrac{1}{2}rw_f - \tfrac{1}{4}t_f + {}^3/_4 t)^2 - \left[60t^2 + w_f t_f \right.$$

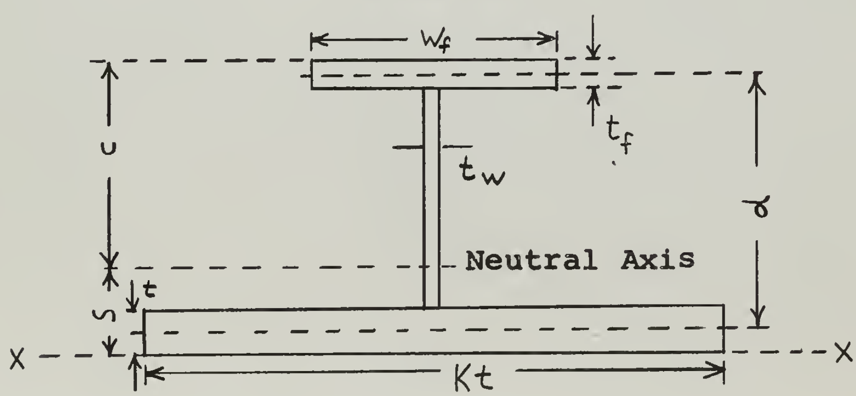
$$\left. \left. + t_w\ (rw_f - \tfrac{1}{2}t_f - \tfrac{1}{2}t) \right] (S)^2 \right\} \qquad (77)$$

Machine computation using the IBM 7090 computer at the M.I.T. Computation Center produced solutions for (76) and (77) for a range of parameters t, r, and w_f. These solutions presented in the form of curves, are found in Appendix B. In all cases these curves give the value of c measured from the neutral axis to the outer surface of the flange. For light plating and relatively large flanges, the extreme distance to the outer fiber may shift to the plate side. The point at which this happens is shown on the curve when appropriate. These curves can be used for rapid determination of moments of inertia and position of neutral axis (and hence of section modulus) of a wide variety of stiffener and plate sizes used in ship construction.

B. Summary of Data and Calculations

The curves in this appendix constitute a summary of the properties of plated tee cross sections. The method of determining the geometrical properties presented by these curves is set forth in detail in Appendix A. For each value of plating thickness between 1/8 inch and one inch (in increments of 1/8 inch) there are plots of c versus I. Upon entering the proper c-I curve with the predetermined values, the r value necessary for the stiffener is found. The value of r just determined and the inertia I are the entering arguments for the r-I plots. The proper r-I curve then determines the required w_f. An example of the use of the curves in the proposed design procedure is found in Appendix C, part III. All properties of the stiffener can then be determined by the relations below.

Figure VI



[illegible]

[illegible]

$$d = rw_f$$

$$t_f = 1/4(t+1)$$

$$t_w = 0.6\ t_f$$

Effective plating width = 60 t'

When using the curves for plating of 1/8, 1/4, and 3/8 inches it must be noted that for w_f greater than 3 1/3, 12, and 24.5 inches respectively, c shifts to the plate side of the neutral axis. The flange width at which this occurs is noted on the appropriate plots. The data presented in the curves of this appendix are valid for any use of plated tee sections having the properties noted above and are not restricted to the proposed design procedure.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

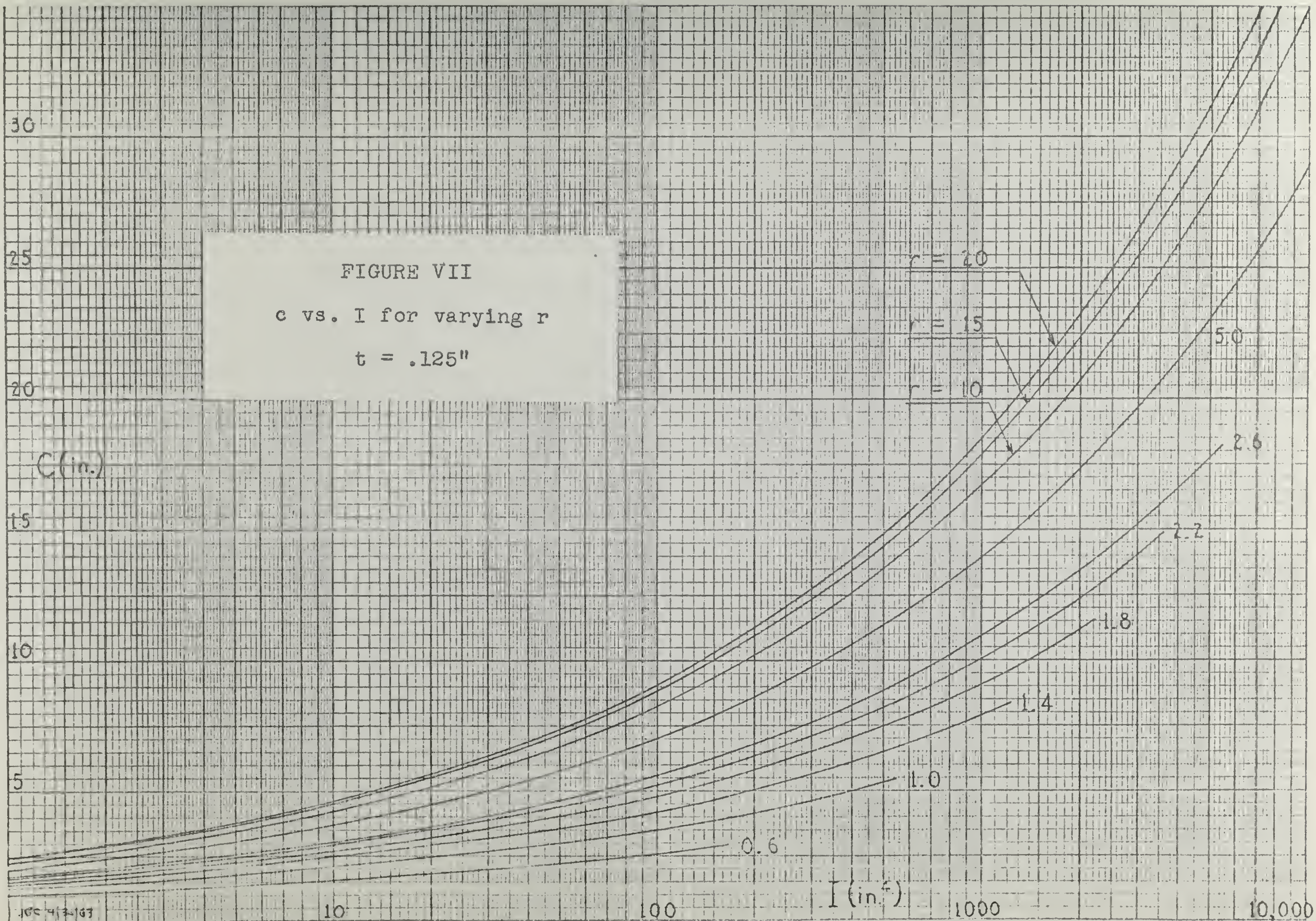


FIGURE VII
c vs. I for varying r
t = .125"
C (in.)
I (in.⁴)
r = 20
r = 15
r = 10
5.0
2.6
2.2
1.8
1.4
1.0
0.6
30
25
20
15
10
5
10
100
1000
10,000

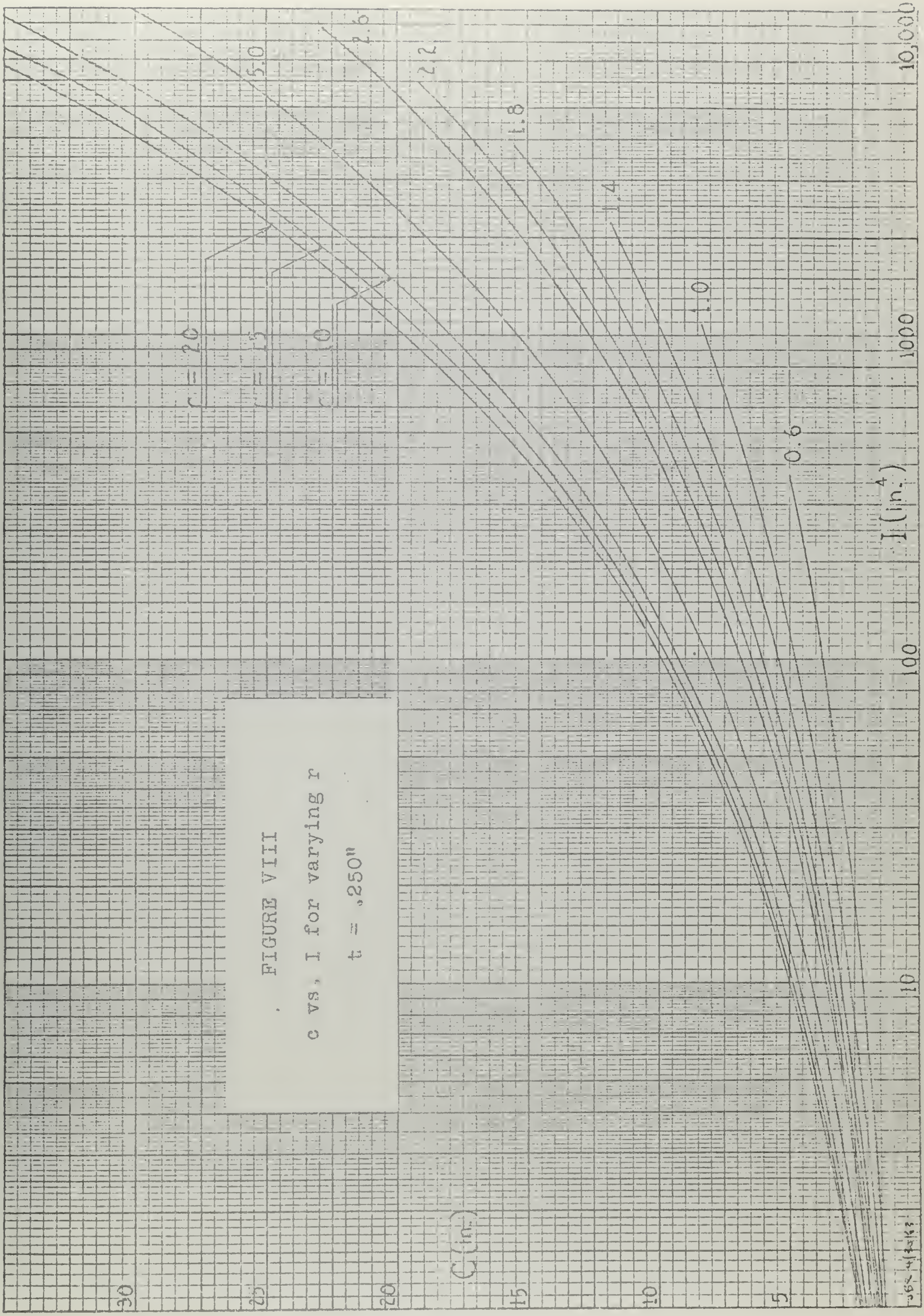


FIGURE VIII

c vs. I for varying r

t = .250"

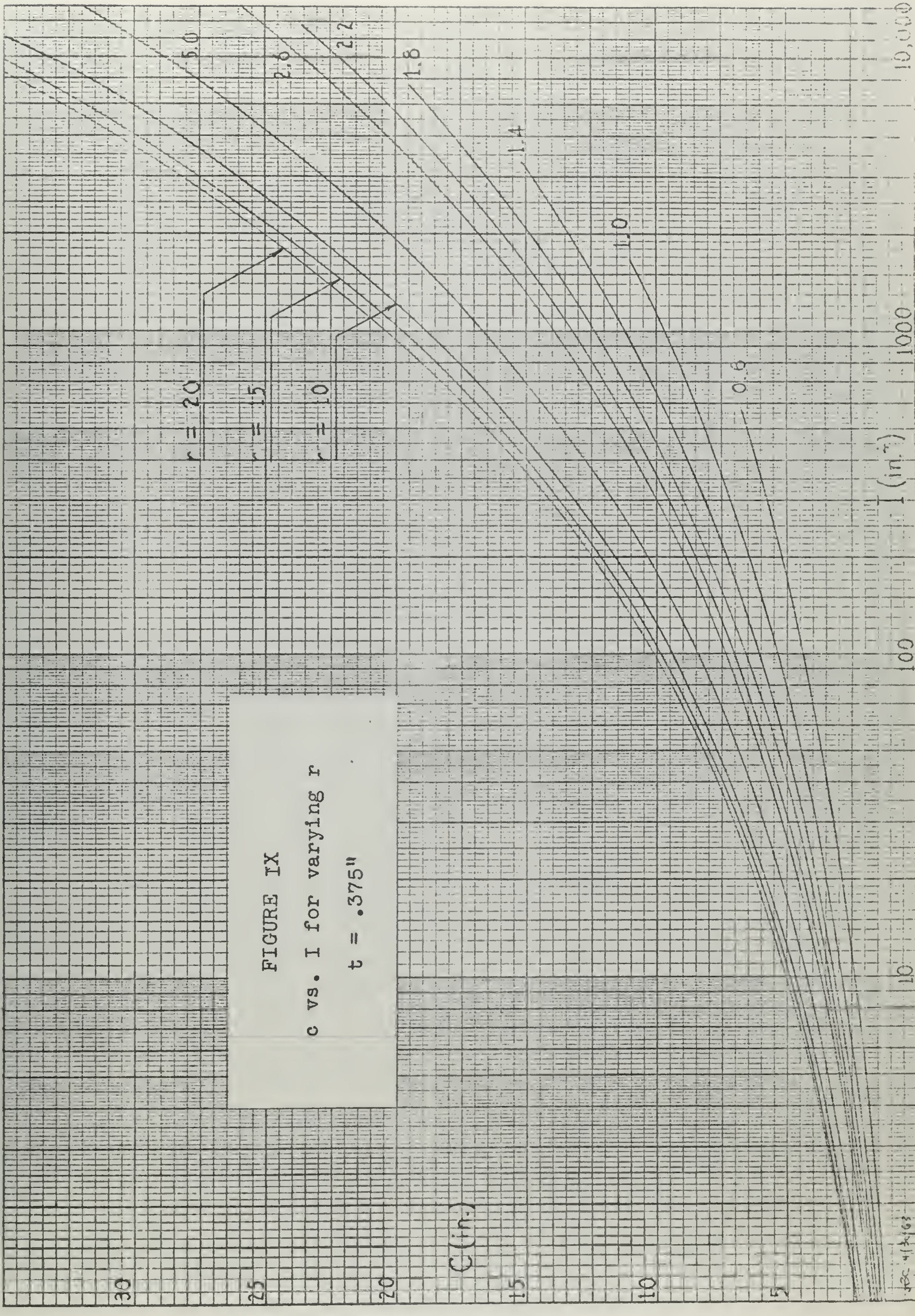


FIGURE IX
c vs. I for varying r
t = .375"

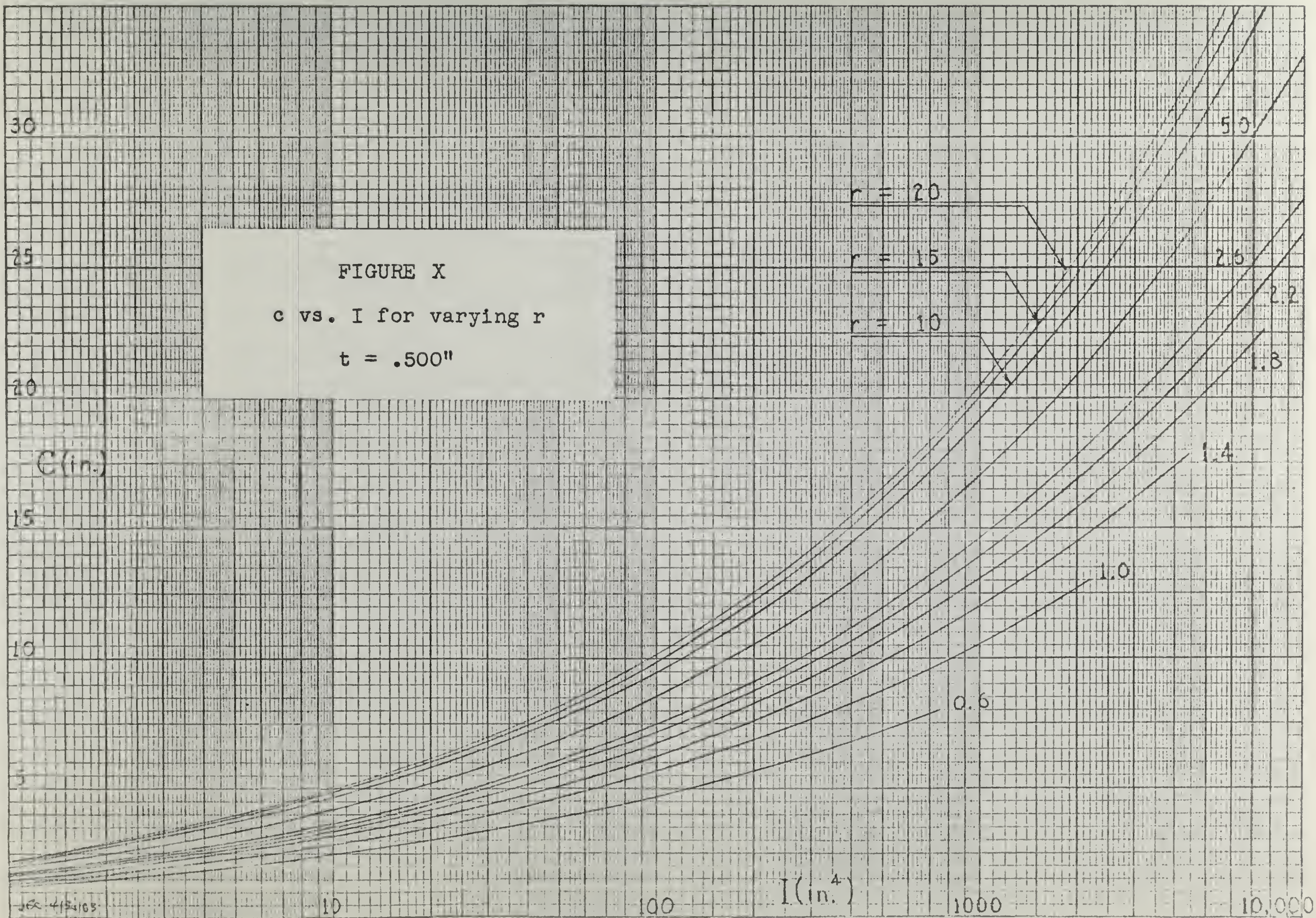


FIGURE X

c vs. I for varying r

t = .500"

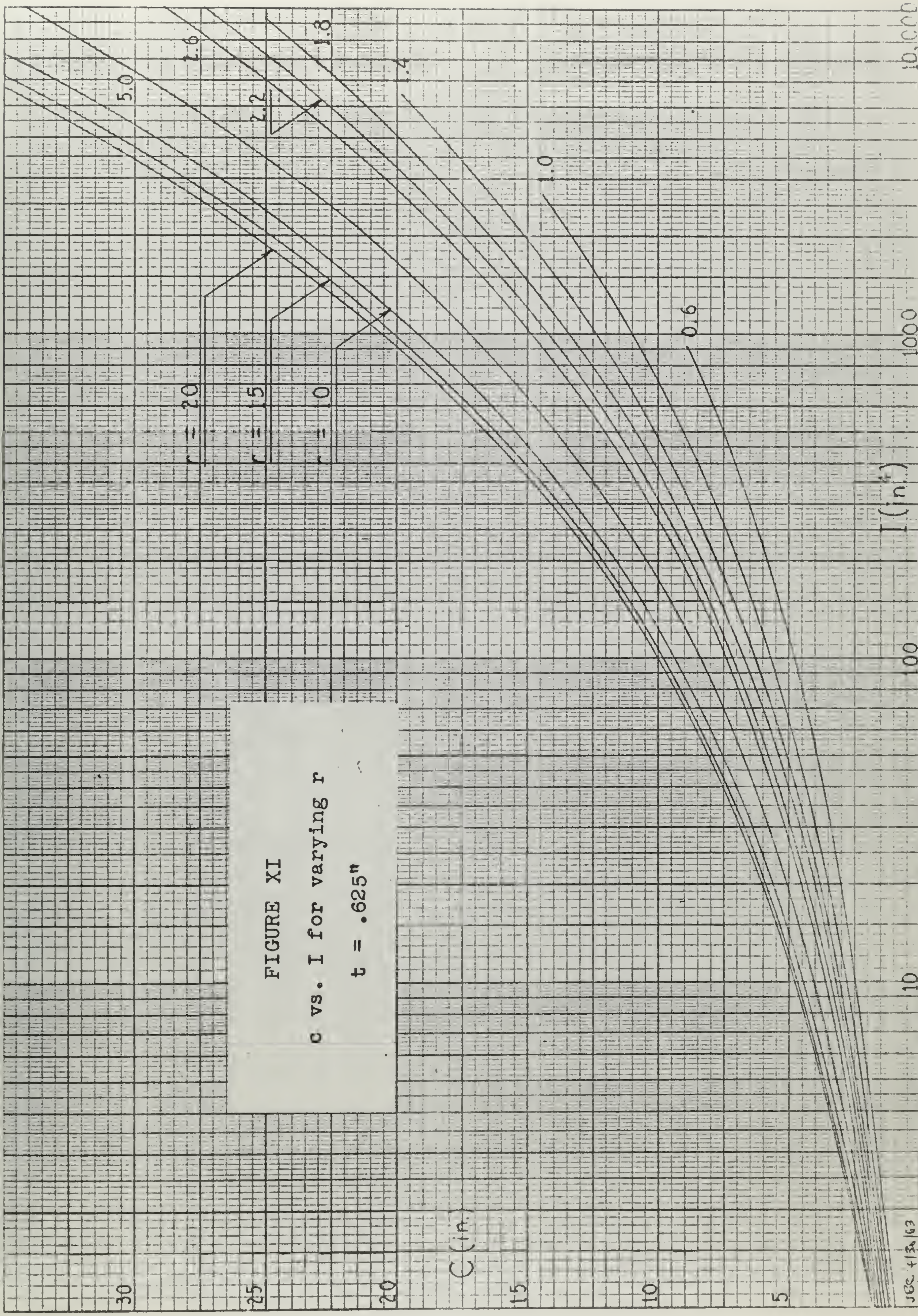


FIGURE XI

c vs. I for varying r

t = .625"

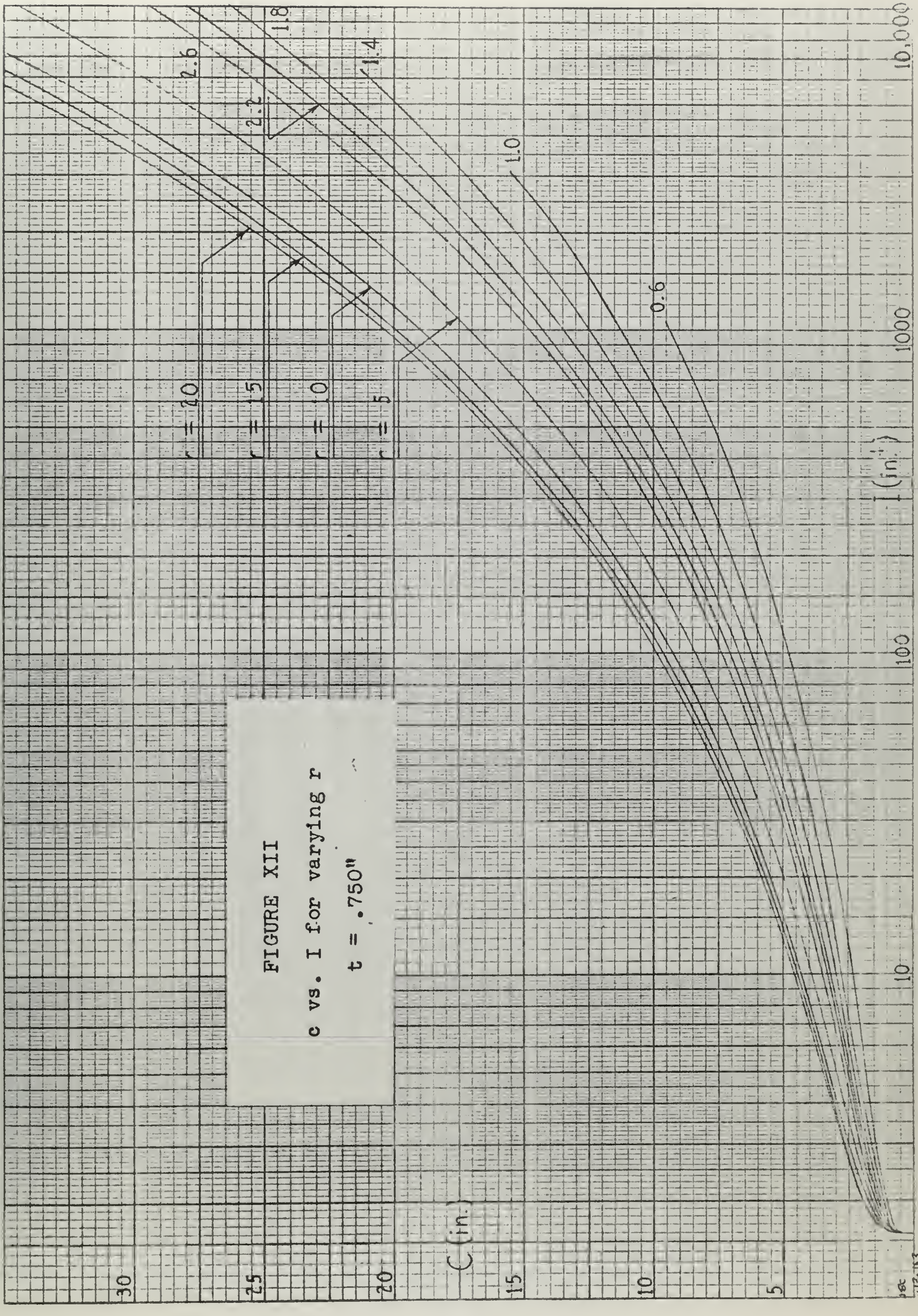


FIGURE XII

c vs. I for varying r

t = .750"

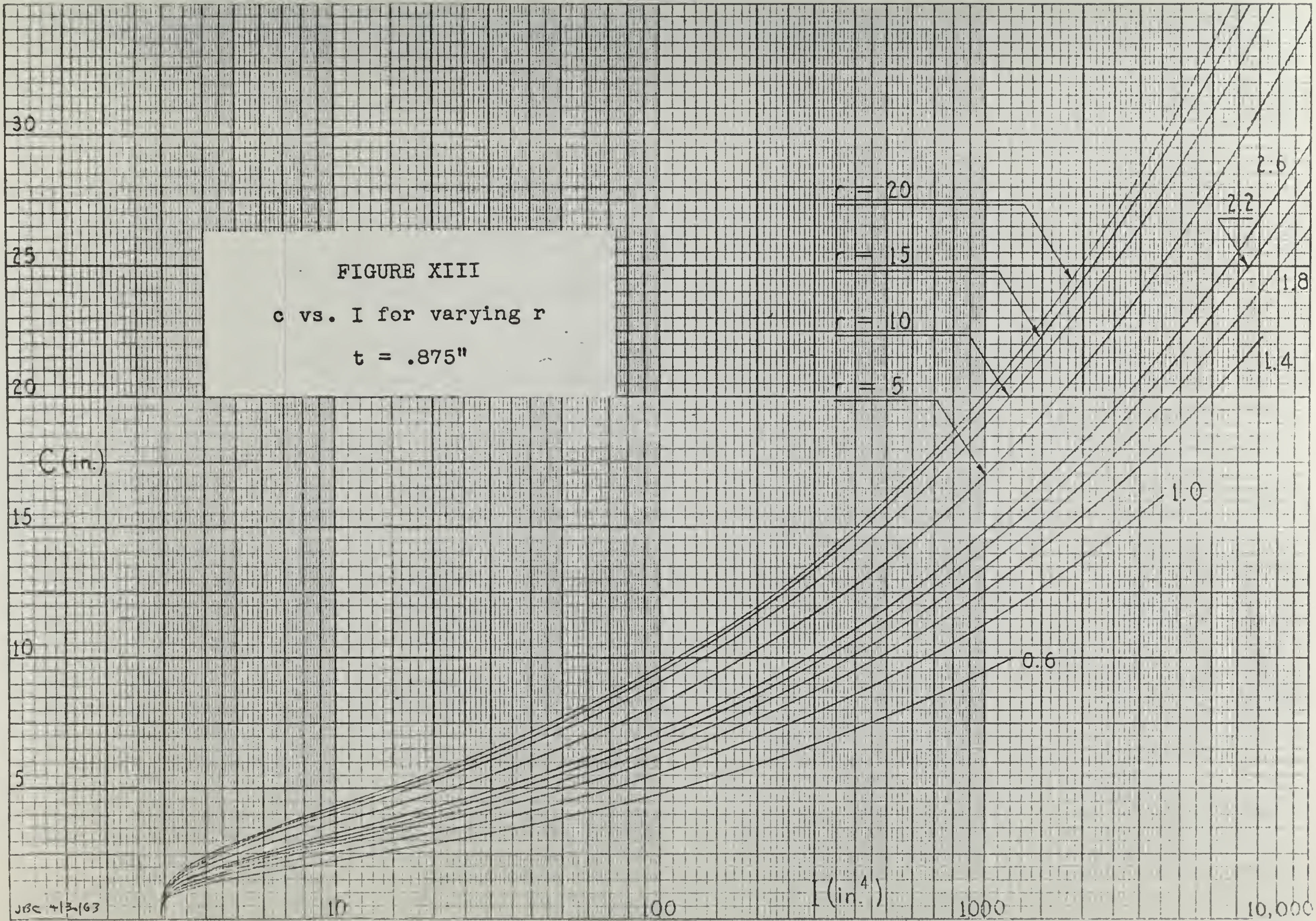


FIGURE XIII

c vs. I for varying r

t = .875"

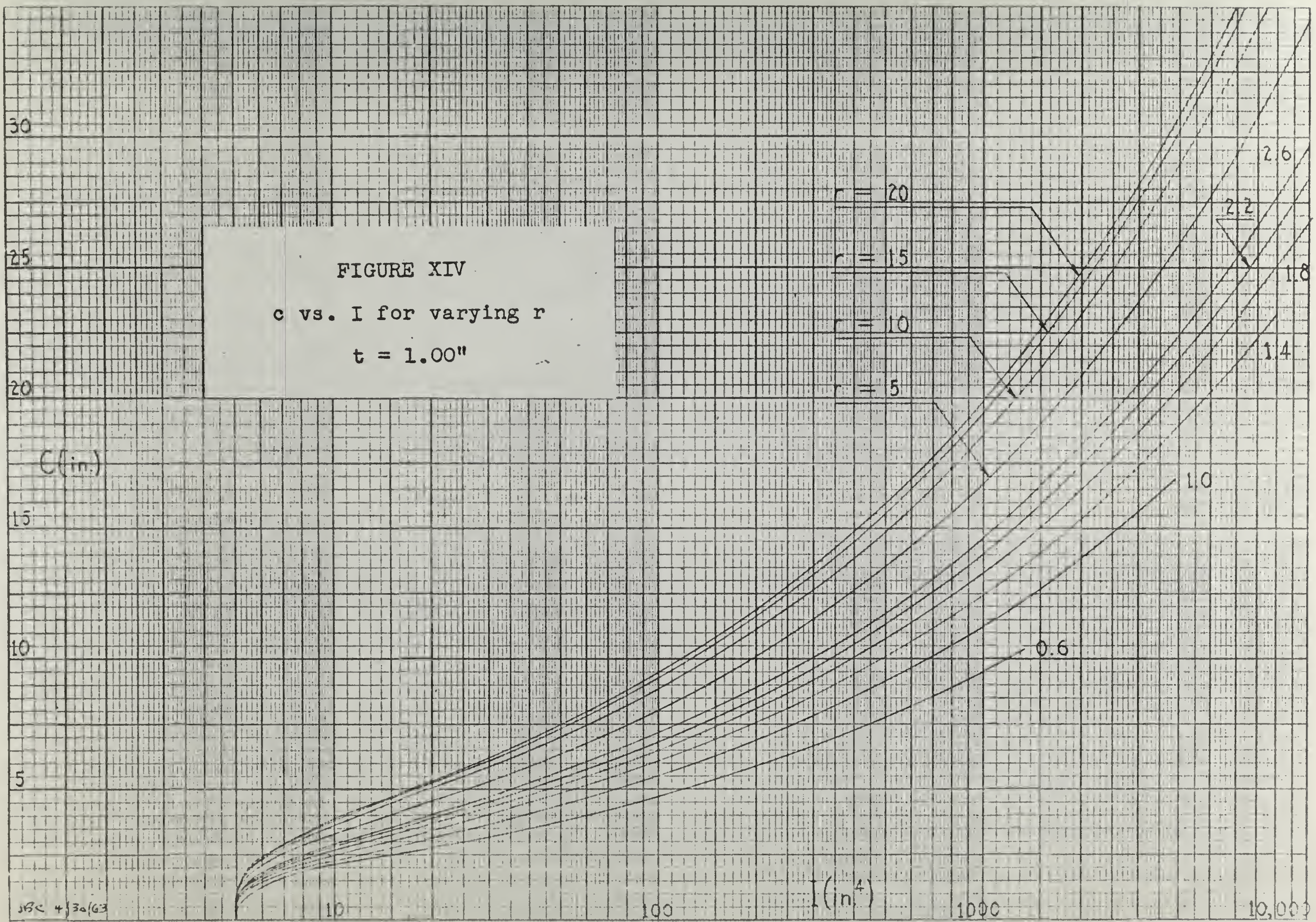


FIGURE XIV
c vs. I for varying r
t = 1.00"
r = 20
r = 15
r = 10
r = 5
2.6
2.2
1.8
1.4
1.0
0.6
C (in.)
I (in^4)
30
25
20
15
10
5
10
100
1000
10,000
JBC 4/30/63

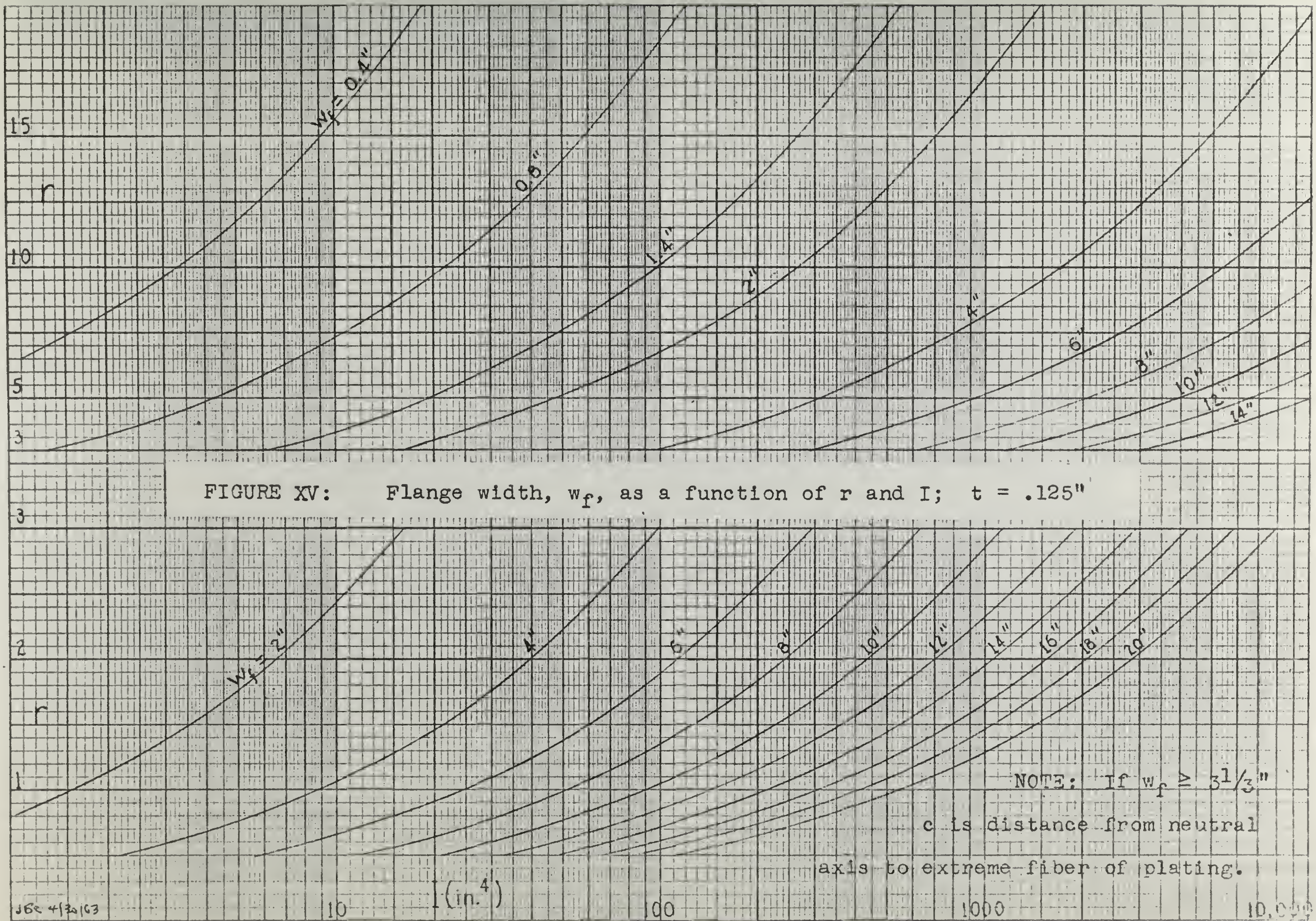


FIGURE XV: Flange width, w_f, as a function of r and I; t = .125"

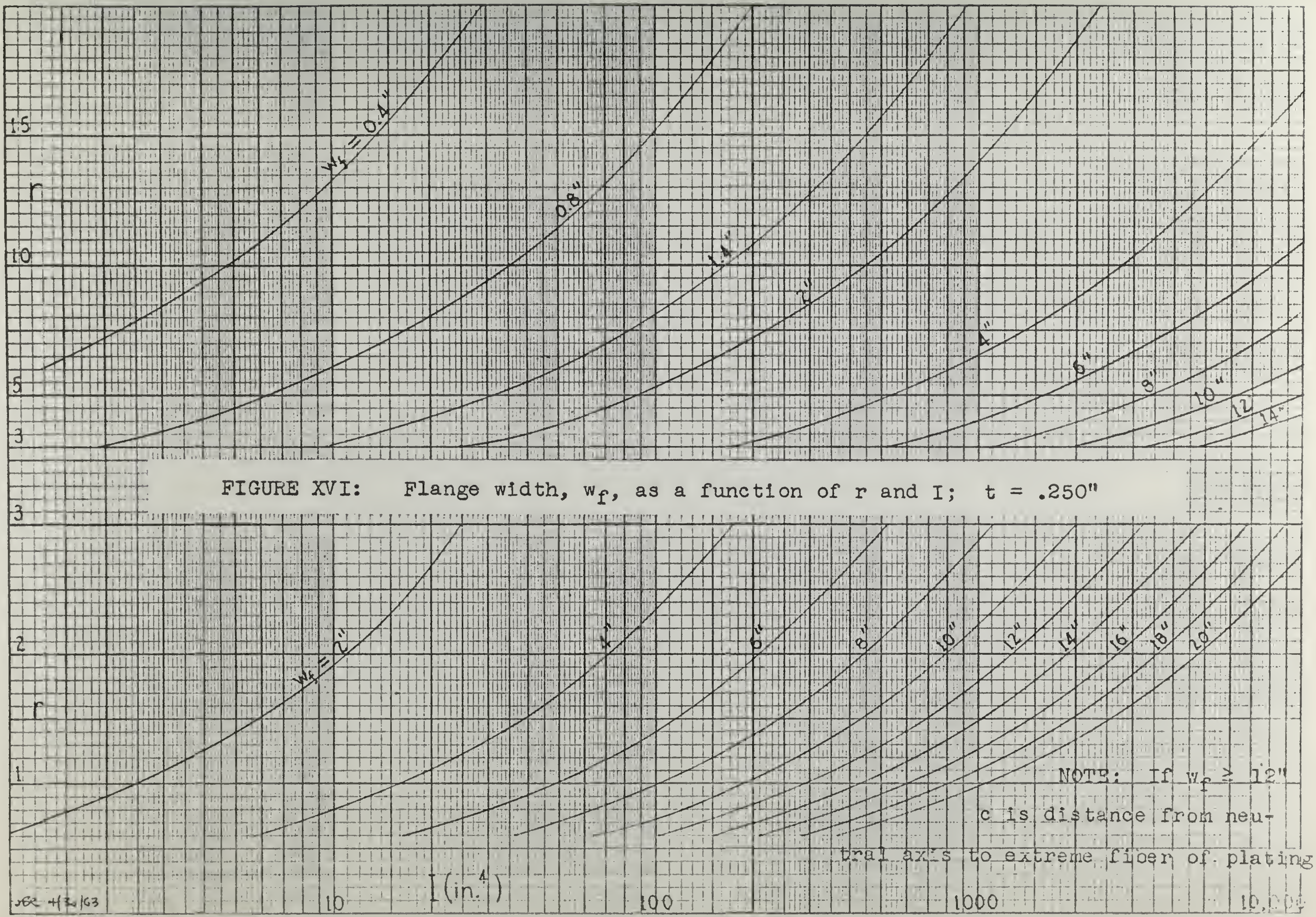


FIGURE XVI: Flange width, w_f, as a function of r and I; t = .250"

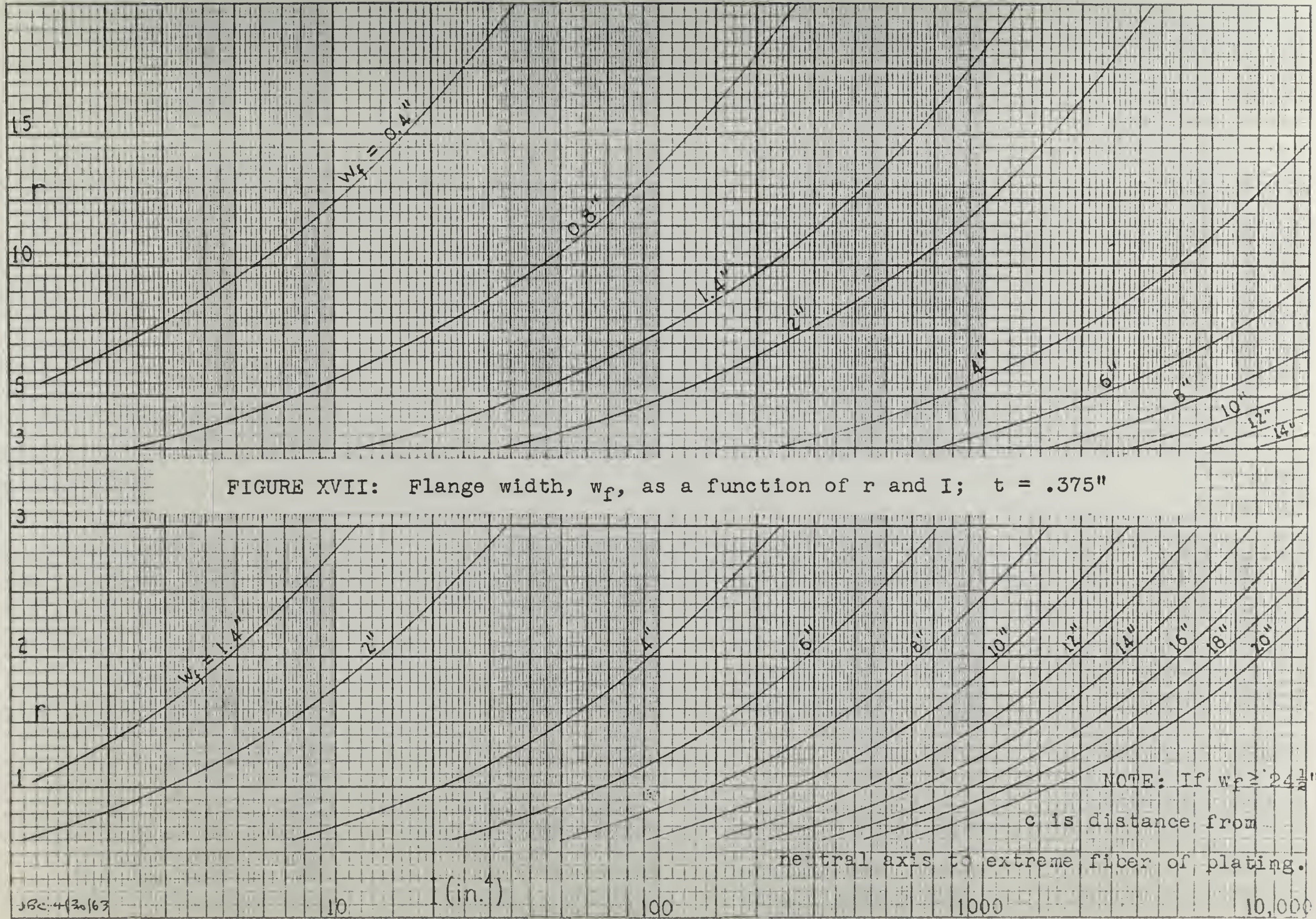


FIGURE XVII: Flange width, w_f, as a function of r and I; t = .375"

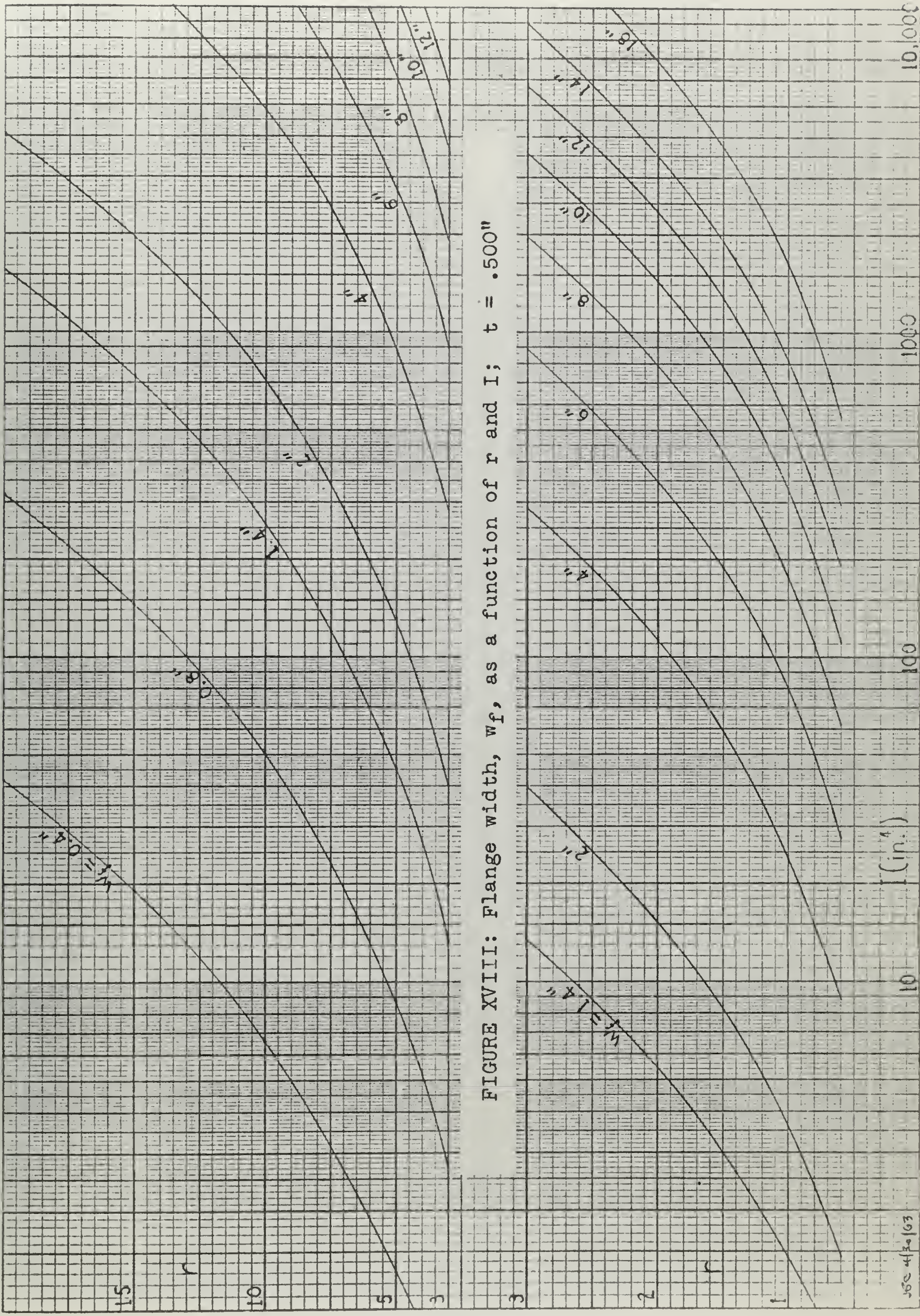


FIGURE XVIII: Flange width, w_f, as a function of r and I; t = .500"

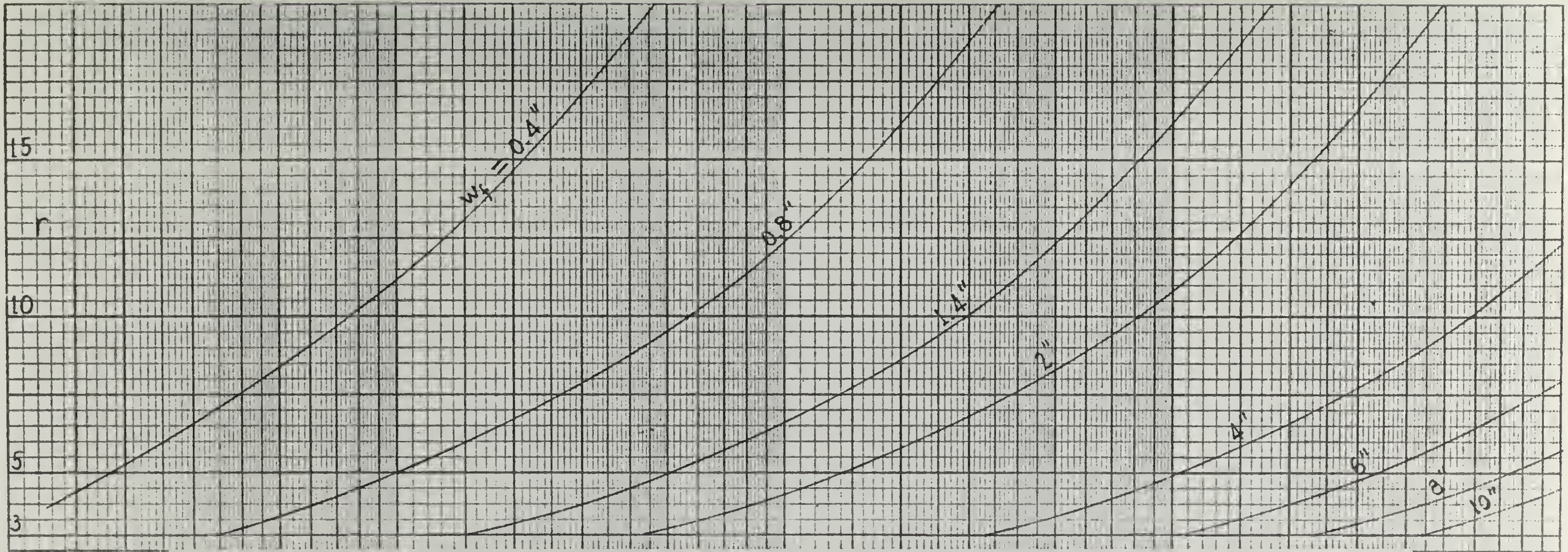
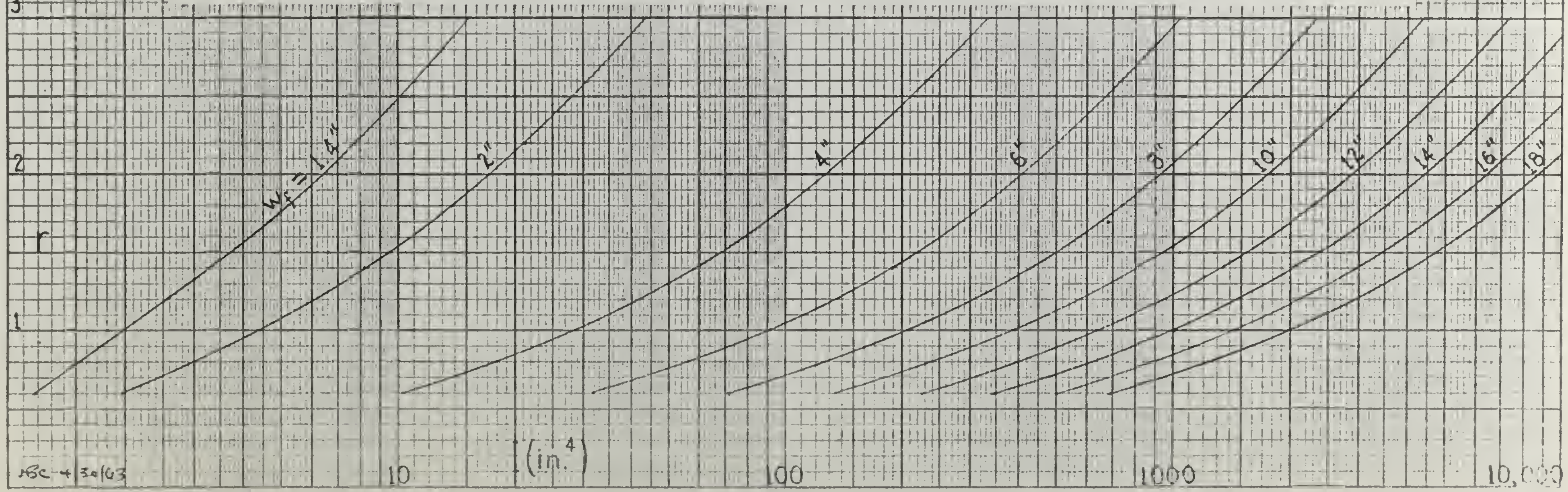


FIGURE XIX: Flange width, w_f, as a function of r and I; t = .625"



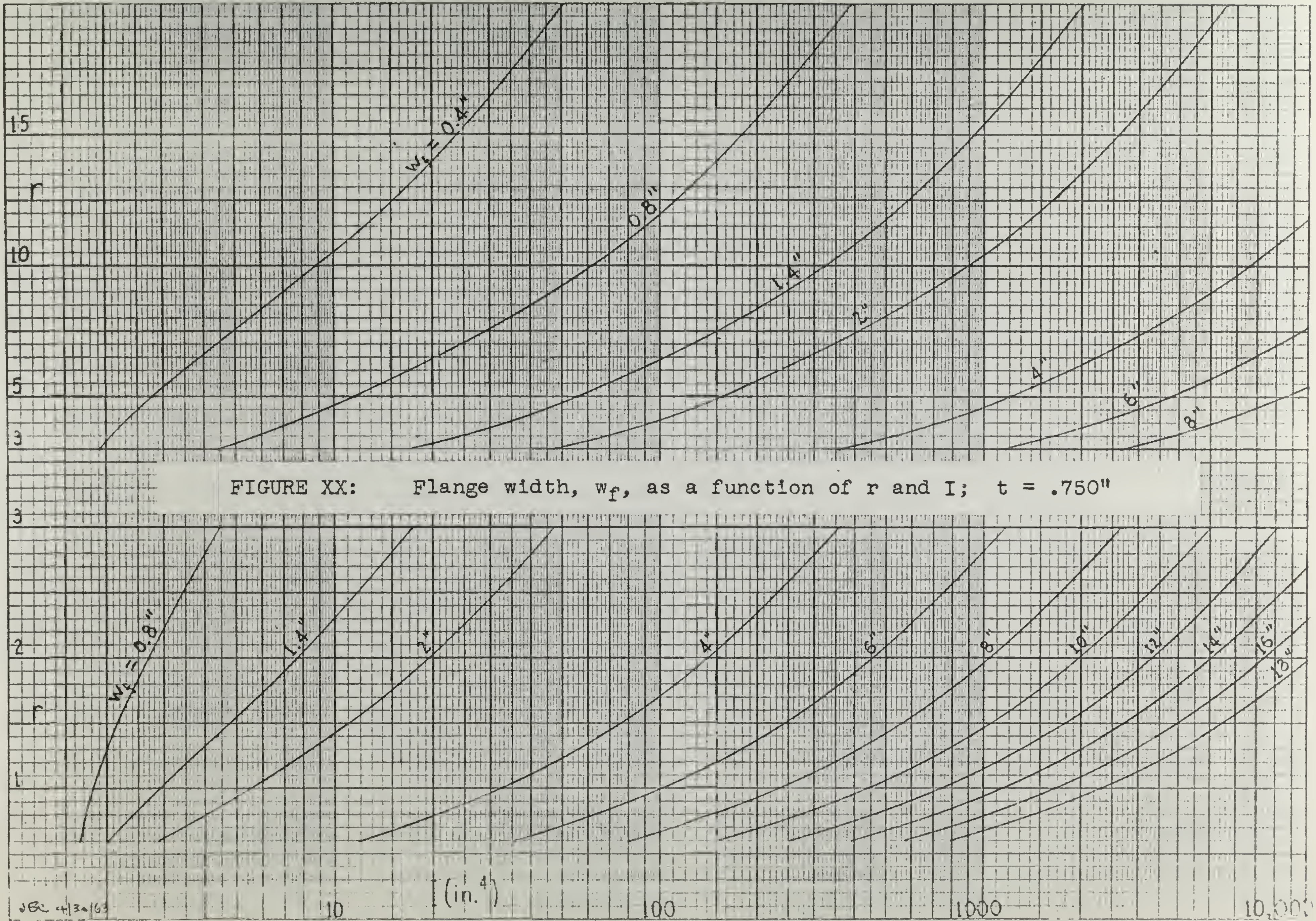


FIGURE XX: Flange width, w_f, as a function of r and I; t = .750"

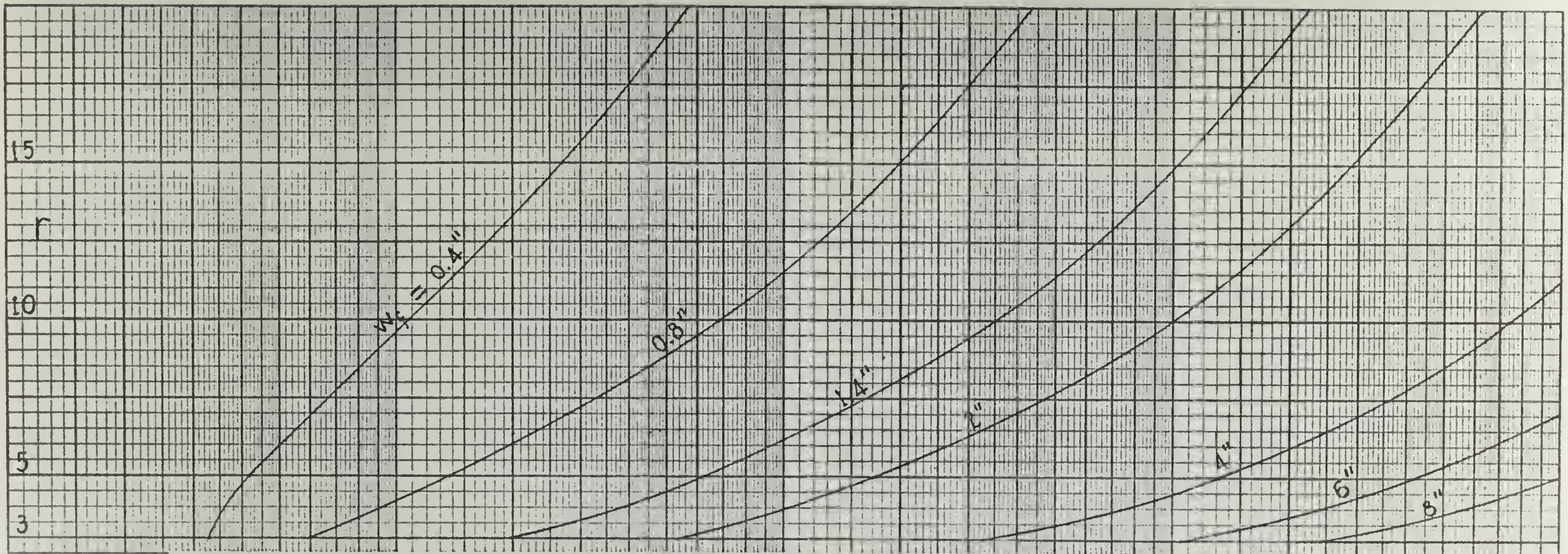
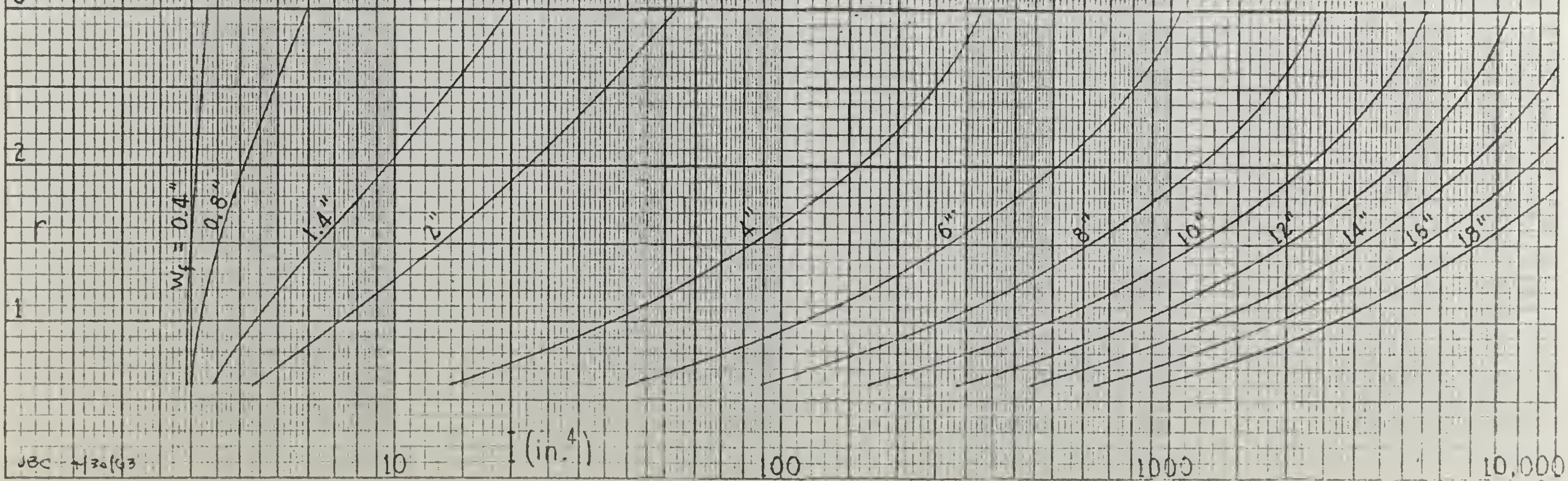


FIGURE XXI: Flange width, w_f, as a function of r and I; t = .875"



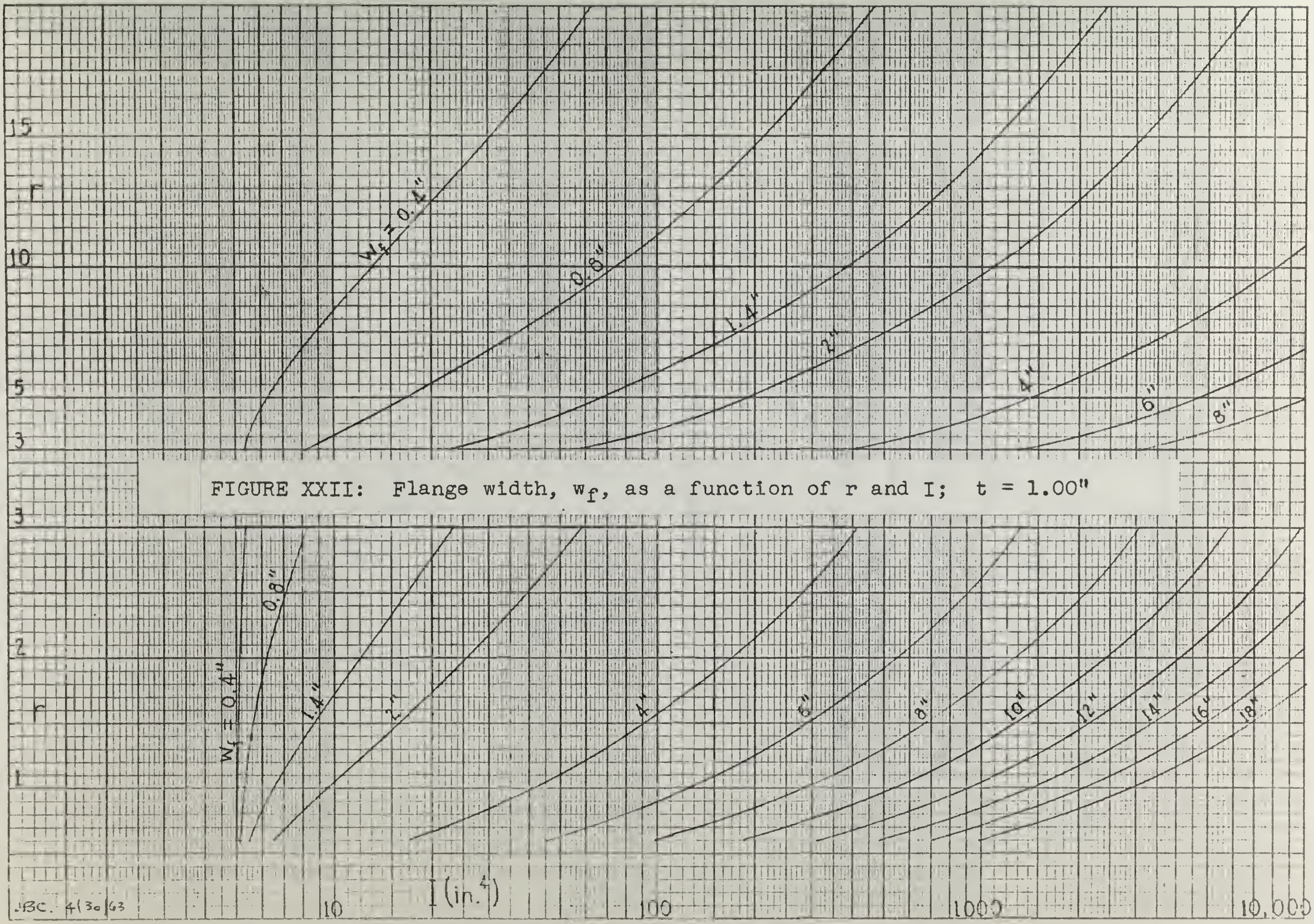


FIGURE XXII: Flange width, w_f, as a function of r and I; t = 1.00"

C. SAMPLE CALCULATIONS

1. To illustrate the differences in weight resulting from the different design procedures, a simply supported, plated tee, single beam will be designed by three different methods.

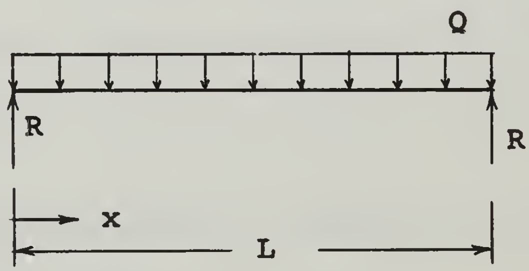
Case 1 - Conventional design for prismatic member.

Case 2 - Minimum weight design with constant curvature.

Case 3 - Minimum weight design with sixth order deflected shape.

The design conditions for all three cases will be:

Figure XXIII



$L = 20$ ft $\qquad Q = 3000$ lb/ft $\qquad t = 0.5"$

$\sigma = 20{,}000$ lb/in^2 $\qquad E = 30 \times 10^6$ lb/in^2

Case 1: $M_x = \frac{Qx}{2}(L - x)$

$$M\,(\max) = \frac{QL^2}{8} = \frac{3000}{8}(20)(240) = 1.80 \times 10^6 \text{ in - lb}$$

then

$$Z = \frac{M}{\sigma} = \frac{1.8 \times 10^6}{20{,}000} = 90 \text{ in}^3$$

Using US Navy Modulus graphs [4], with $t = 0.5"$, the lightest stiffener plating combination is:

T - 16" x 7" x 50# (36.22 lb/ft)

Then, the total stiffener weight will be:

$$W = 36.22 \times 20 = 724.4 \text{ lb.}$$

To insure that all beams are compared on the same basis, determine the deflection for this case, and use it as the basis of the following designs. From [4], for the stiffener selected,

$$I = 1110 \text{ in}^4$$

$$w_o = \frac{5\ Q\ L^4}{384\ EI} = \frac{5 \times 3000 \times 20 \times (240)^3}{384 \times 30 \times 10^6 \times 1110} = 0.324"$$

This value of w_o will be used to design the minimum weight beams.

Case 2: Using the development of Appendix A (2)

$$M = Q\left[\frac{L^2}{8} - \frac{x^2}{2}\right]$$

where x is measured from the center of the beam. Thus, the moment expression is:

$$M = \frac{3000 \times 12}{2}\left[\frac{400}{4} - x^2\right] = 12{,}000\ (100 - x^2) \text{ in-lb.}$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

where x is in feet. For a single beam differentiate (12) twice to get

$$W_x'' = w_o \left[8/L^2 \right] = \frac{8 \times .324}{(240)^2} = .45 \times 10^{-4}$$

Then

$$C = \frac{\sigma}{E W_x''} = \frac{20,000}{30 \times 10^6 \times .45 \times 10^{-4}} = 14.81''.$$

and so

$$I_x = \frac{M_x}{E W_x''} = \frac{12,000 \ (100 - x^2)}{30 \times 10^6 \times .45 \times 10^{-4}}$$

$$I_x = 8.89 \quad (100 - x^2)$$

Now, compute I_x for various locations along the length. Then use Figure X to get r, and Figure XVIII to get W_f

TABLE I

x(ft)	$I(in^4)$	r	W_f (in)	d (in)	A_s (in^2)
0	889	3.3	5.5	18.13	6.04
2.5	834	3.6	5.0	18.0	5.82
5	666	4.6	3.95	18.15	5.46
7.5	389	14.0	0.6	8.4	2.02
10	--	--	0	11.198	2.42

The area of the stiffener at the various cross sections was computed as follows, using equations (74), (74a), and (74b) from Appendix A.

$$A_s = d_w t_w + w_F t_f$$

$$t_f = 1/4 \ (t + 1) = 1/4(3/2) = 0.375''$$

$$tw = 0.6 \ t_f = 0.225''$$

$$d_w = d - 1/2 \ t - 1/2 \ t_f = d - 0.438''$$

So

$$A_s = 0.225\ (d - 0.438) + 0.375\ w_f$$

For shear considerations, the area cannot go to zero at the ends. Using the strain-energy failure criterion,

$$\tau_y = 0.62\ \sigma_y$$

and if we use the same factor of safety as we did with σ, then

$$\tau\,(\text{allow}) = 0.62\ (20{,}000) = 12{,}400\ \text{lb/in}^2$$

The maximum shear will occur at the ends where

$$R = 1/2\ QL = \frac{3000 \times 20}{2} = 30{,}000\ \text{lb.}$$

Therefore, the required area to resist the average shear at the ends will be:

$$A = \frac{R}{\tau} = \frac{30{,}000}{12{,}400} = 2.42\ \text{in}^2$$

Thus, the area at x = 10' must be 2.42 in^2. Since the web thickness is 0.225", the required depth must be

$$d_w = \frac{A}{.225} = \frac{2.42}{.225} = 10.76"$$

thus $\quad d = 10.76 + .438 = 11.198"$

These values are entered in the appropriate parts of the preceding table.

Using Simpson's Rule to integrate the area over the half length of the beam gives

Total Vol. (stiffener) = 1014.8 in.^{3}

With γ (steel) = 0.2833 lb/in^3

the total weight of the stiffener is:

W = 0.2833 x 1014.8 = 287.5 lb.

Case 3: Using the development of Appendix A (5) for a single beam we have

$$W''_x = W_o \left[\frac{8}{L^2} + \frac{96\ x^2}{L^4} - \frac{960\ x^4}{L^6} \right]$$

by differentiating (69) twice. Substituting for Wo and L gives:

$$W''_x = \frac{.324}{144 \times (20)^2} \left[8 + \frac{96\ x^2}{(20)^2} - \frac{960\ x^4}{(20)^4} \right]$$

$$W_x'' = 5.63 \times 10^{-6} \left[8 + .24\ x^2 - .006\ x^4 \right]$$

Also

$$I_x = \frac{c_x}{\sigma} M_x = 12{,}000 \frac{c_x}{\sigma} (100 - x^2)$$

$$I_x = 0.6\ c_x \ (100 - x^2)$$

Using (2)

$$C_x = \frac{\sigma}{Ew_x''} = \frac{20{,}000}{30 \times 10^6 \times 5.63 \times 10^{-6} (8 + .24\ x^2 - .006\ x^4)}$$

$$C_x = \frac{118.5}{8 + .24\ x^2 - 006\ x^4}$$

Now, compute C_x and I_x for various locations along the length.

Again, as in Case 2, use Figure X to get r and Figure XVIII for w_f.

Table II

x(ft)	C(in)	I(in^4)	r	Wf(in)	d(in)	A_s(in^2)
0	14.81	889	3.3	5.6	18.47	6.15
2.5	12.80	720	2.30	7.0	16.10	6.14
5	11.58	521	2.20	6.4	14.09	5.47
7.5	46.9	1230	$\to \infty$	$\to 0$	11.20	2.42
10	-4.24	0	-	-	11.20	2.42

The areas were computed in the same manner as for Case 2, and shear requirements again fixed the required area at x = 10 ft. At x = 7.5 ft, design requirements exceeded the limits of the curves.

Integrating by Simpson's Rule again gives:

[illegible]

Total Vol. (Stiffener) = 1075.0 in^3.

Then the total weight is

W = 0.2833 x 1075.0 = 304.5 lbs.

2. In the case of beams with rectangular cross-sections, it can be shown that the redundant internal reactions have no effect on the total weight of the grillage. From strain energy considerations, considering only bending with small deformations, we know that

$$U = 1/2\ E \int_0^L I_x\ w_x''\ dx$$

for a single beam. For the case of constant curvature, this becomes

$$U = 1/2\ E\ w_x'' \int_0^L I_x\ dx \tag{78}$$

Now, from the Principle of Least Work,

$$\frac{\partial U}{\partial R} = 0$$

where R is any redundant reaction. Also

$$A = {}^{12}\!/_{h^2}\ I$$

for rectangular sections, so that

$$\text{Weight} = \gamma \int_0^L A_x\ dx = \frac{12\gamma}{h^2} \int_0^L I_x\ dx$$

where h = Constant along the length.

Rearranging gives

$$\int_0^L I_x\ dx = \frac{W\ h^2}{12\ \gamma} \tag{79}$$

Substituting (79) into (78) gives

$$U = 1/2\ E\ w_x'' \left(\frac{Wh^2}{12\gamma} \right) = \frac{E\ w_x''\ h^2\ W}{24\ \gamma}$$

and then

$$\frac{\partial U}{\partial R} = \frac{E\ w_x''\ h^2}{24\ \gamma} \cdot \frac{\partial W}{\partial R} = 0$$

so

$$\frac{\partial W}{\partial R} = 0$$

and thus the weight of any beam with rectangular cross section and constant depth, deforming with constant curvature under a lateral load, is independent of the value of any redundant reaction. Since the weight of any one beam is independent of its redundant reactions, the weight of the grillage, which is merely the sum of the individual beam weights, is also independent of the reactions.

To further illustrate this point, consider a 3 x 3 grillage made up of beams of rectangular cross sections. Then, from the development of Appendix A (4), equation (66) gives

$$W = - \frac{48\ \gamma\ E\ w_o}{\sigma^2} \left\{ \sum_j \left[\frac{4j^2}{(q+1)^2} - 1 \right] \left[\frac{a\ Q_x\ (p+1)}{24} + 1/16 \sum_i R_j^i - \frac{1}{2(p+1)^2} \sum_i R_j^i \langle -i \rangle^2 \right] + \sum_i \left[\frac{4i^2}{(p+1)^2} - 1 \right] \left[\frac{b\ Q_y\ (q+1)}{24} + {}^1/_{16} \sum_j R_i^j - \frac{1}{2(q+1)^2} \sum_j R_i^j \langle -j \rangle^2 \right] \right\}$$

For the 3 x 3 grid,

$$p = 3 \qquad\qquad q = 3$$

$$i = -1,\ o,\ 1 \qquad\qquad j = -1,\ o,\ 1$$

Making these substitutions, we get

$$W = -\frac{48\,\gamma E\, w_o}{\sigma^2}\left\{ \sum_{j=-1}^{1}\left[\frac{4j^2}{16} - 1\right]\left[\frac{a\,Q_x}{6} + \frac{1}{16}\sum_{i=-1}^{1} R_j^i - \frac{1}{32}\sum_{i=-1}^{1} R_j^i \langle -i \rangle^2\right] \right.$$

$$\left. + \sum_{i=-1}^{1}\left[\frac{4i^2}{16} - 1\right]\left[\frac{b\,Q_y}{6} + \frac{1}{16}\sum_{j=-1}^{1} R_i^j - \frac{1}{32}\sum_{j=-1}^{1} R_j^i \langle -j \rangle^2\right]\right\}$$

Expanding

$$W = -\frac{48\,\gamma E\, w_o}{\sigma^2}\left(\begin{aligned} &\left(-\frac{12}{16}\right)\left[\frac{a\,Q_x}{6} + \frac{1}{16}\left(R^{-1}_{-1} + R^{o}_{-1} + R^{1}_{-1}\right) - \frac{1}{32}R^{-1}_{-1}\right] \\ &+(-1)\left[\frac{a\,Q_x}{6} + \frac{1}{16}\left(R^{-1}_{o} + R^{o}_{o} + R^{1}_{o}\right) - \frac{1}{32}R^{-1}_{o}\right] \\ &+\left(-\frac{12}{16}\right)\left[\frac{a\,Q_x}{6} + \frac{1}{16}\left(R^{-1}_{1} + R^{o}_{1} + R^{1}_{1}\right) - \frac{1}{32}R^{-1}_{1}\right] \end{aligned}\right) \text{ i beams}$$

$$-\frac{48\,\gamma E\, w_o}{\sigma^2}\left(\begin{aligned} &\left(-\frac{12}{16}\right)\left[\frac{b\,Q_y}{6} + \frac{1}{16}\left(R^{-1}_{-1} + R^{o}_{-1} + R^{1}_{-1}\right) - \frac{1}{32}R^{-1}_{-1}\right] \\ &+(-1)\left[\frac{b\,Q_y}{6} + \frac{1}{16}\left(R^{-1}_{o} + R^{o}_{o} + R^{1}_{o}\right) - \frac{1}{32}R^{-1}_{o}\right] \\ &+\left(-\frac{12}{16}\right)\left[\frac{b\,Q_y}{6} + \frac{1}{16}\left(R^{-1}_{1} + R^{o}_{1} + R^{1}_{1}\right) - \frac{1}{32}R^{-1}_{1}\right] \end{aligned}\right) \text{ j beams}$$

But, by symmetry, the i = -1 longitudinal is identical with the i = 1 longitudinal, and the j = -1 transverse is identical

with j = 1 transverse. Also, by symmetry $R_j^i = R_j^{-i}$ and $R_i^j = R_i^{-j}$. With these conditions the weight equation becomes

$$W = \frac{48\,\gamma\,E\,w_o}{\sigma^2}\left[\frac{5\,a\,Q_x}{12} + \frac{5\,b\,Q_y}{12}\right]$$

$$+ \frac{48\,\gamma\,E\,w_o}{\sigma^2}\left\{\begin{array}{ll} \frac{12}{8}\left[\frac{1}{16}\left(2\,R_{-1}^{-1} + R_{-1}^{o}\right) - \frac{1}{32}\,R_{-1}^{-1}\right] & \text{i beams} \\ \quad + \frac{1}{16}\left(2\,R_o^{-1} + R_o^{o}\right) - \frac{1}{32}\,R_o^{-1} & \\ + \frac{12}{8}\left[\frac{1}{16}\left(2\,R_{-1}^{-1} + R_{-1}^{o}\right) - \frac{1}{32}\,R_{-1}^{-1}\right] & \text{j beams} \\ + \frac{1}{16}\left(2\,R_o^{-1} + R_o^{o}\right) - \frac{1}{32}\,R_o^{-1} & \end{array}\right\}$$

But, by equal and opposite reactions

$$R_{-1}^{-1}\Big|_i^j = -\,R_{-1}^{-1}\Big|_j^i$$

$$R_o^{-1}\Big|_i^j = -\,R_{-1}^{o}\Big|_j^i \qquad \text{etc.}$$

so that all the R terms cancel, and we are left with

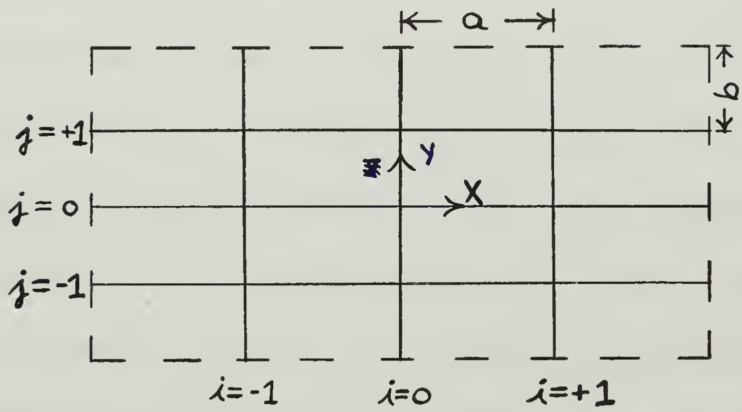
$$W = \frac{\gamma\,E\,w_o}{\sigma^2}\left[20\,a\,Q_x + 20\,b\,Q_y\right]$$

which is independent of the values of the internal reactions.

3. Example of design of ship type, simply supported grillage by the proposed inverse method.

Figure XXIV

Grillage Dimensions



a = 6 ft
b = 3 ft
H = 20 ft
p = 3
q = 3

From equations (5) and (4).

$$Q_y = 2\,\frac{pb}{4} = \frac{2(20)\ 3(.64)}{4} = 1920 \text{ lbs/ft}$$

$$Q_x = 2\,\frac{pb}{4}\left(2 - \frac{b}{a}\right) = 1920\left(\frac{3}{2}\right) = 2880 \text{ lbs/ft}$$

Selection of plate thickness depends primarily on H and a/b. There are many published criteria for plate selection such as [4] and [7]. Entering sheet six of [4] leads to a plate thickness of $\frac{1}{2}$ inch for this example.

Equations (26) and (27) are then evaluated to find the required inertia variations.

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$$I_x^j = \frac{-(6)^2(4)^2}{8\ E\ W_o} \left[\frac{9(4)^2}{4(3j)^2-9(4)^2}\right] \Bigg\{ 2880 \left(\frac{36(4)^2}{8} - \frac{x^2}{2}\right) + \tfrac{1}{2}\sum_i R_j^i (x + 12)$$
$$- R_j^i \langle x - 6\ i \rangle \Bigg\} \text{ in-ft.}^3 \qquad (80)$$

where all distances along the beam are in feet, E in psi, and W_o in inches.

It has been suggested that the selection of beam intersection reactions is a design decision and does not affect the total weight of the stiffeners. For this example a constant reaction of 2000 lbs. acting downward (positive sense) on the y beams is selected arbitrarily to demonstrate procedure. Typical maximum deflections are of the order of $\frac{1}{360}$ (maximum span) as noted in [4].

$$W_o = \frac{1}{360}(24 \times 12) = .8 \text{ in.}$$

Using, also, E = 30 million psi for medium steel reduces (80) for the individual beams to,

$$I_x^o = .005184 \left[1440(144-x^2) - 3000(x+12) - R_o^i \langle x-6i \rangle\right] \text{in.}^4 \qquad (81)$$

Similarly:

$$I_x^{\pm 1} = .006912 \left[1440(144-x^2) - 3000(x+12) - R_{\pm 1}^i \langle x-6i \rangle\right] \text{in.}^4 \qquad (82)$$

$$I_y^o = .001296 \left[960\ (36-y^2) + 3000(y+6) - R_o^j \langle y-3j \rangle\right] \text{in.}^4$$

$$I_y^{\pm 1} = .001728 \left[960(36-y^2) + 3000(y+6) - R_{\pm 1}^j \langle y-3j \rangle\right] \text{in.}^4 \qquad (83)$$

The value of c for any one of the above beams is found from (28).

$$\frac{\sigma}{E w_o} = C_x^j \left[\frac{4j^2}{(q+1)^2} - 1\right]\left[\frac{-8}{a^2(p+1)^2}\right] = C_y^i \left[\frac{4i^2}{(p+1)^2} - 1\right]\left[\frac{-8}{b^2(q+1)^2}\right] \tag{84}$$

Using values of E and W_δ from above and taking σ = 27,000 psi as a reasonable design stress leads to

$$\frac{\sigma}{E\, W_\delta} = \frac{27{,}000}{30 \times 10^6 (.8)} = .001125/\text{in.}$$

Therefore $$C_x^o = \frac{.001125}{\left[\frac{4(o)}{4^2} - 1\right]\left[\frac{-8}{(72)^2 (4)^2}\right]} = 11.66 \text{ in.}$$

Similarly:

$$C_x^{\pm 1} = 15.55 \text{ in.}$$

$$C_y^o = 2.92 \text{ in.}$$

$$C_y^{\pm 1} = 3.89 \text{ in.}$$

A sample inertia calculation to demonstrate use of the singularity notation is made here for X = + 8 feet on the $y = +1$ beam.

$$I_x^{+1} = .006912 \left\{1440(144-64) - 3000(8+12) - (-)2000\left[8-6(-1)\right] - (-)2000\left[8-6(o)\right] - (-)2000\left[8-6(+1)\right]\right\}$$

$$= .006912 \left[1440(80) - 3000(20) + 2000(14+8+2)\right]$$

$$= .006912 \ (103{,}200)$$

$$= 713.3 \text{ in.}^4$$

[illegible]

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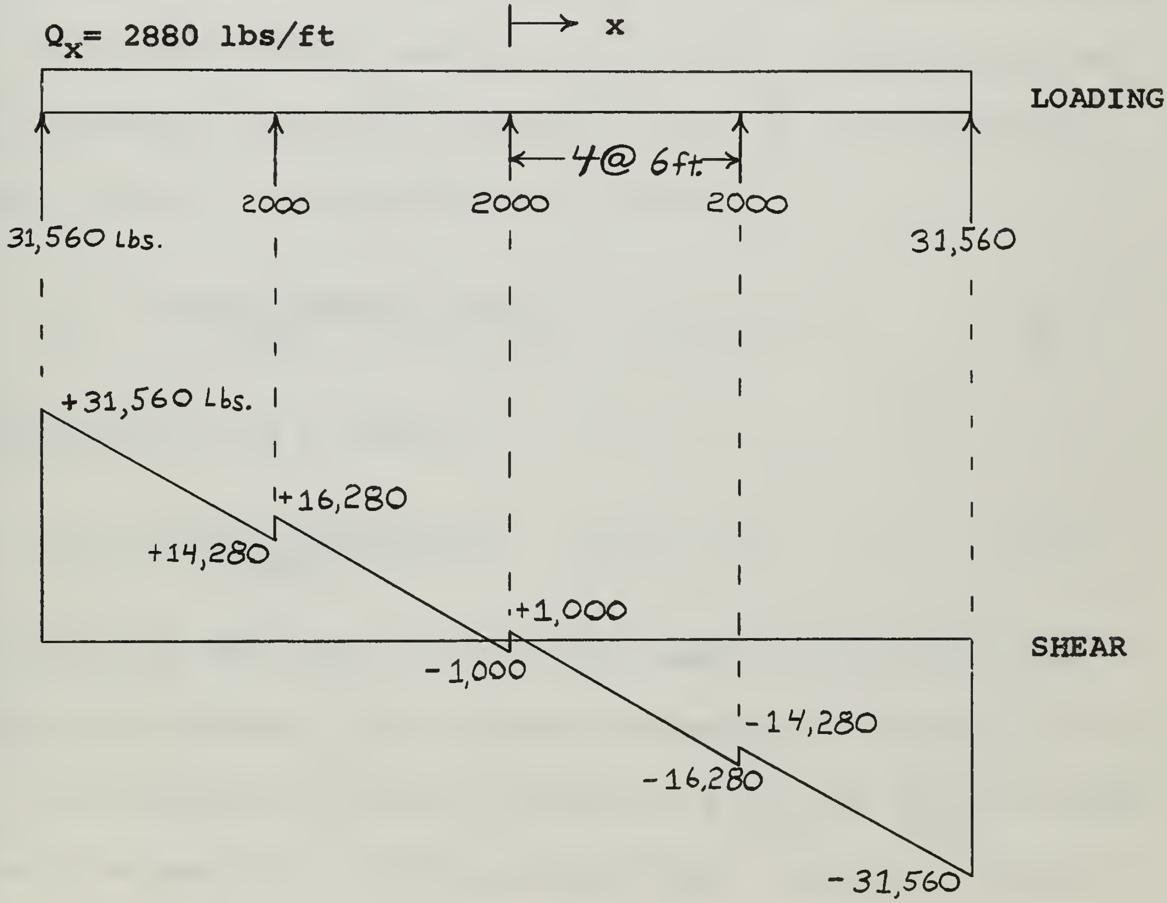
The use of (80) through (83) will clearly give zero required inertia at the respective beam ends. Shear requirements must then be examined to design the beam ends. Figure XXIV shows the loading and shear diagrams for the X beams.

FIGURE XXIV

Load and shear for X beams



The shearing force at any point written as a function of distance from the left end of the beam, is:

$$31{,}560-2880\ (X+12),\ -12 < X < -6 \tag{85}$$

Assuming that all shear is resisted by the web of depth d leads to the value of d_s required by shear at any point along the beam.

$$31{,}560 - 2880(X + 12) = t_w\ d_s\ \tau \tag{86}$$

where τ is taken as 16,740 psi for mild steel. The design shear stress is set at 62% of the design yield stress according to the maximum strain energy criteria [8]. Substitution and rearrangement leads to an expression for d_s as required by shear for any X between -12 and -6 feet.

$$d_s = \frac{31{,}560-2880(x + 12)}{3776},\quad - 12 < X < -6 \tag{87}$$

Similarly for the y beams:

$$d_s = \frac{14{,}520 - 1920(y+6)}{3776},\quad -6 < y < -3 \tag{88}$$

Solutions of (87) and (88) near the ends of the respective beams are included in the columns headed d_s in tables III to VI.

Calculations of various values of I and the use of corresponding values of c in conjunction with figures X and XVI lead to the complete grillage design which is presented in tables III to VI..The areas in these tables are of stiffener

[illegible]

only, and are included to allow calculation of stiffener weight for comparison with conventional design.

TABLE III

Summary of y^o beam

$C_y^o = 2.92$ in.

y (ft)	$I_y^o(in^4)$	r	$W_f(in)$	d (in)	$d_s(in)$	Area(in^2)
± 6.0	0	—	(2.4)	—	3.86	1.67
± 5.5	9.1	1.20	2.4	2.88	3.60	1.61
± 5.0	15.1	.72	4.4	3.17	3.34	2.30
± 4.0	32.6	.43	8.0	3.44	2.84	3.68
± 3.0	42.8	.40	9.4	3.76		4.27
± 2.0	52.8	.36	11.0	3.96		4.92
± 1.0	57.8	.35	11.2	3.92		4.98
0	60.3	.34	11.5	3.91		5.09

Depth will be controlled by shear from y= ± 4.5 feet to the ends. Flange width will be taken as 2.4 inches for y greater than ± 5.5. feet

TABLE IV

Summary of $y^{\pm 1}$ beams

$C_y^{\pm 1} = 3.89$ in.

y(ft)	$I_y^{\pm 1}$ (in^4)	r	W_f(in)	d(in)	d_s(in)	Area(in^2)
± 6.0	0	-	(1.5)	-	3.86	1.33
± 5.5	12.1	2.50	1.5	3.75	3.60	1.31
± 5.0	20.1	1.35	3.0	4.05	3.34	1.94
± 4.0	43.5	.68	6.6	4.49	2.84	3.39
± 3.0	57.0	.57	8.2	4.67		3.98
± 2.0	70.4	.53	9.0	4.77		4.35
± 1.0	77.1	.51	9.7	4.95		4.65
0	80.4	.50	10.0	5.00		4.78

Depth will be controlled by shear only in the last 1/2 foot. Flange width will be held constant the last 1/2 foot.

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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

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TABLE V

Summary of X^o beams

$C^o_x = 11.66$ in.

X(ft)	$I^o_x(in^4)$	r	$W_f(in)$	d(in)	$d_s(in)$	Area (in^2)
± 12	0	-	--	--	8.4	2.04
± 11.5	79.9	high	--	--	8.0	2.65
± 11	156.1	21.00	.65	13.6	7.6	3.21
± 10	297.4	4.76	3.0	14.3	6.8	4.25
± 8	535.0	2.36	6.1	14.4	5.4	5.44
± 6	764.7	1.63	9.1	14.8		6.65
± 4	851.8	1.50	10.2	15.3		7.17
± 2	931.0	1.36	11.4	15.5		7.68
0	950.5	1.25	11.7	15.8		7.85

Depth will be controlled by shear in the last 1/2 foot and flange width held constant the last foot.

A sketch of the depth and flange width profile for the beam is shown in Figure XXVI

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FIGURE XXVI

PROFILES OF CENTRAL X STIFFENER

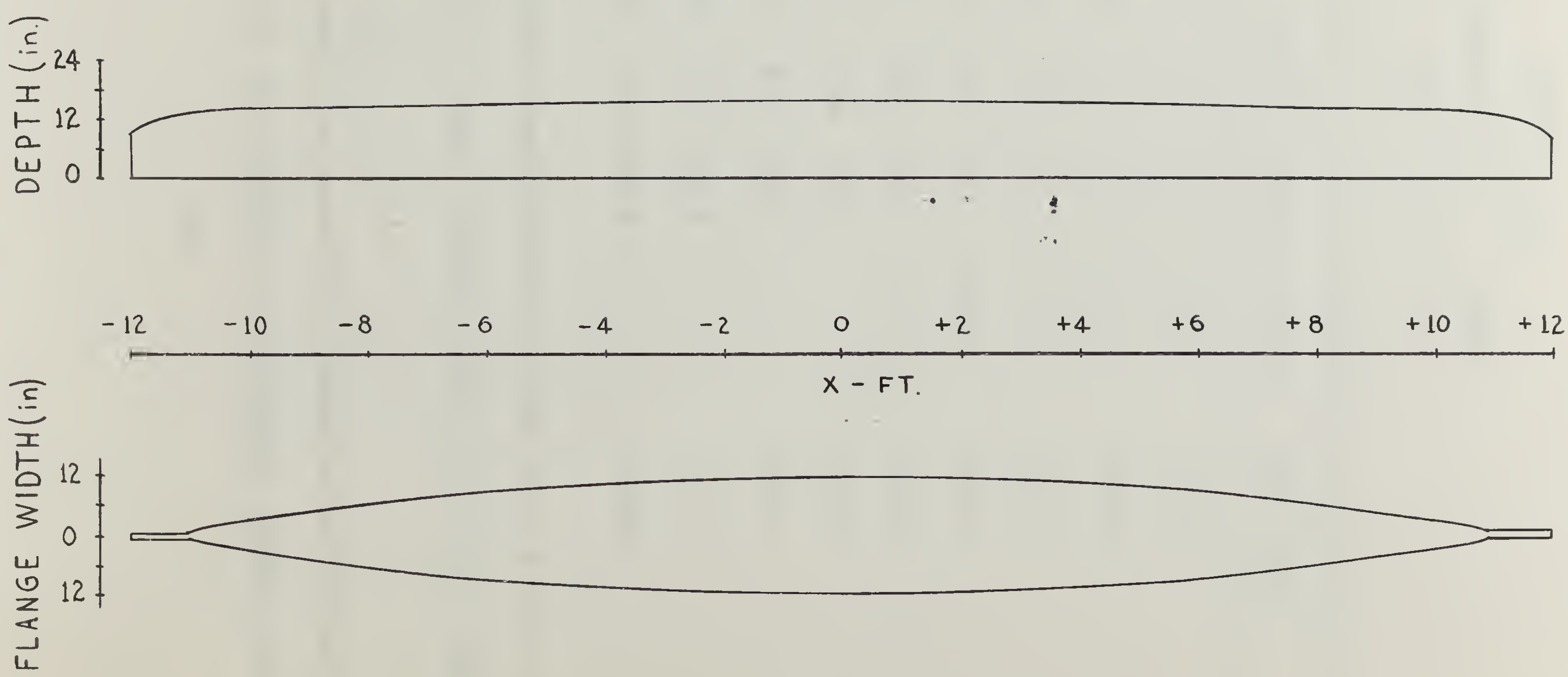

TABLE VI

Summary of $X^{\pm 1}$ beams

$C_x^{\pm 1} = 15.55$ in.

X(ft)	$I_x^{\pm 1}$ (in^4)	r	W_f(in)	d(in)	d_s(in)	Area (in^2)
$\pm$ 12	0	--	--	--	8.4	1.99
$\pm$ 11.5	106.6	high	--	--	8.0	3.56
$\pm$ 11.	208.2	high	--	--	7.6	4.12
$\pm$ 10.	396.5	high	--	--	6.8	4.71
$\pm$ 8.	713.3	5.3	3.6	19.1	5.4	5.56
$\pm$ 6	1,019.6	3.5	5.6	19.6		6.42
$\pm$ 4	1,135.8	3.0	6.5	19.5		6.74
$\pm$ 2	1,241.4	2.60	7.6	19.8		7.21
0	1,267.4	2.54	7.8	19.8		7.28

Depth will be controlled by shear in the last ½ foot and faired in to 19.1 at $\pm$ 8 feet. Flange width will fair down to .5 inches.

Integration of the cross sectional areas of each beam by Simpson's Rule leads to the weight of stiffeners shown below.

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]					[illegible]
[illegible]	[illegible]			[illegible]	[illegible]	[illegible]
[illegible]	[illegible]			[illegible]	[illegible]	
[illegible]	[illegible]			[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]		[illegible]	[illegible]	[illegible]	[illegible]	

[illegible]

[illegible]

[illegible]

TABLE VII

Stiffener weights

Stiffener	Weight
I_y^o	159.0 lbs
$I_y^{\pm 1}$	144.8
I_x^o	496.0
$I_x^{\pm 1}$	486.1

Total Stiffener weight = 1886.8 lbs.

4. Using the data sheets of [6] and [4], a simply supported 3 x 3 grillage under the uniform pressure will be designed by a conventional method to compare the stiffener weights with the stiffener weights obtained by the minimum weight design procedure. For purposes of comparison, the design parameters are the same as those used for the minimum weight design.

As pointed out in Section I, conventional design techniques require trial and error procedures. In this case, it was necessary to make several trials with assumed values of inertia to obtain the required central deflection of 0.8". The required inertias necessary to give this deflection are:

$$I_x = 922 \text{ in.}^4 \qquad I_y = 51.9 \text{ in.}^4$$

using Figure G.1 [6]. Then, from Figures G.7 and G.8 [6], the maximum bending moments in the grillage are,

$$M_x = -2.71 \times 10^6 \text{ in-lb}$$

$$M_y = -579,220 \text{ in-lb}$$

where M_y, the moment in the light beams, is strongly influenced by local bending between adjacent beam intersections.

With the design stress of

$$\sigma = 27,000 \text{ psi}$$

the required section moduli are:

$$Z_x = \frac{Mx}{\sigma} = 100.3 \text{ in}^3$$

$$Z_y = \frac{My}{\sigma} = 21.4 \text{ in}^3$$

Using the Modulus graphs [4], the plate-stiffener combinations most closely matching the required Z and I are:

x Beams: T - 16" x 8½" x 58# (40.8 lb/ft.)

$$Z_x = 108 \text{ in}^3 \quad I_x = 1190 \text{ in}^3.$$

y Beams: ½ - 14" I x 6-3/4" x 30# (15.0 lb/ft.)

$$Z_y = 21.4 \text{ in}^3 \quad I_y = 127 \text{ in.}^4$$

The total weight of the stiffeners is then

$$W = 3 \ (W_x + W_L)$$

$$W = 3 \ (40.8 \times 24 + 15.0 \times 12)$$

$$W \doteq 3480 \text{ lb.}$$

D. Bibliography

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